**MATH 3132 Tutorial 3**

**1.** Set up, but do **NOT** evaluate, a double iterated integral for the value of the surface integral

$$\iint_S (x^2 + z)\, dS$$

where $S$ is that part of $z = y^2$ bounded by the planes $x = 1$, $x = 0$, and $z = 1$.

**2.** Evaluate the surface integral

$$\iint_S (3x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\, dS$$

where $S$ is that part of $z = x^2 + y^2$ below $z = 2$, and $\hat{\mathbf{n}}$ is the unit lower normal to the surface.

**3.** Set ip, but do **NOT** evaluate, a double iterated integral for the value of the surface integral

$$\iint_S z\, dS$$

where $S$ is that part of the surface $z = 4 - x^2 - y^2$ between the planes $z = 2$ and $z = 0$.

**4.** Evaluate the surface integral

$$\iint_S [x\hat{\mathbf{i}} + y\hat{\mathbf{j}} - (1 + xz)\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where $S$ is that part of $z = x^2 + y^2 - 4$ below the $xy$-plane, and $\hat{\mathbf{n}}$ is the unit upper normal to the surface.

**5.** Evaluate the surface integral

$$\iint_S xy\, dS$$

where $S$ is that part of the surface $x + y + z = 1$ in the first octant.

**6.** Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$$

where $S$ is that part of the surface $z = 4 - x^2 - y^2$ above the $xy$-plane, $\hat{\mathbf{n}}$ is the unit upper normal, and $\mathbf{F} = xz\hat{\mathbf{i}} + yz\hat{\mathbf{j}}$.

**7.** Evaluate the surface integral

$$\iint_S (x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\, dS$$

where $S$ is the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$, and $\hat{\mathbf{n}}$ is the unit normal to $S$ with positive $z$-component. ($a > 0$ is the radius of the hemisphere.)

**8.** Evaluate the surface integral

$$\iint_S xy^2\, dS$$

where $S$ is that part of the surface $z = x^2$ bounded by the planes $z = 4$, $y = 0$, and $y = 1$.

**9.** Evaluate the surface integral

$$\iint_S x\,dS$$

where $S$ is that part of the surface $y = z + x^2$ in the first octant to the left of the plane $y = 4$.

**Answers**: **1.** $\displaystyle\int_0^1 \int_{-1}^1 (x^2+y^2)\sqrt{1+4y^2}\,dy\,dx$ **2.** $10\pi$ **3.** $\displaystyle\int_0^{2\pi} \int_{\sqrt{2}}^2 r(4-r^2)\sqrt{1+4r^2}\,dr\,d\theta$
**4.** $-20\pi$ **5.** $\sqrt{3}/24$ **6.** $64\pi/3$ **7.** $\pi a^4/2$ **8.** $0$
**9.** $(18^{3/2} - 2\sqrt{2})/3 + (20 \cdot 18^{3/2} - 6 \cdot 18^{5/2})/480$