MATH 3132 Tutorial 4

1. Evaluate the surface integral

$$\oiint_S [x^2\hat{\mathbf{i}} - y^2\hat{\mathbf{j}} - z(x^2+y^2)\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where S is the surface $x^2+y^2+z^2 = a^2$ ($a > 0$ is a constant), and $\hat{\mathbf{n}}$ is the unit inward pointing normal to the surface.

2. Evaluate the surface integral

$$\oiint_S (x^2 z\hat{\mathbf{i}} - y\hat{\mathbf{j}} + 3z^2\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\, dS$$

where S is the surface enclosing the volume bounded by $z = \sqrt{x^2+y^2}$ and $z = 1$, and $\hat{\mathbf{n}}$ is the unit outer normal to S.

3. Evaluate the surface integral

$$\oiint_S (x^3\hat{\mathbf{i}} + y^3\hat{\mathbf{j}} + xz\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\, dS$$

where S is the surface enclosing the volume bounded by $z = x^2+y^2$ and $z = 4 - x^2 - y^2$, and $\hat{\mathbf{n}}$ is the unit inward pointing normal to S.

4. Evaluate the surface integral

$$\oiint_S (x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} - xy^3\hat{\mathbf{k}}) \cdot \hat{\mathbf{n}}\, dS$$

where S is the surface enclosing the volume in the first octant bounded by $x+y+z = 1$, $x = 0$, $y = 0$, and z=0, and $\hat{\mathbf{n}}$ is the unit inward pointing normal to S.

5. Evaluate the surface integral

$$\iint_S [x\hat{\mathbf{i}} + y\hat{\mathbf{j}} - (1+xz)\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where S is that part of the surface $z = 4 - (x^2+y^2)$ above the xy-plane, and $\hat{\mathbf{n}}$ is the unit upward pointing normal to S.

6. Evaluate the surface integral

$$\iint_S [(x^4+y)\hat{\mathbf{i}} + (z+y^3)\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where S is that part of the surface $z = x^2+y^2-4$ below $z = 1$, and $\hat{\mathbf{n}}$ is the unit downward pointing normal to S.

7. Evaluate the surface integral

$$\iint_S [(xy^2+z)\hat{\mathbf{i}} + yz^2\hat{\mathbf{j}} - xy^3\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where S is that part of $x^2+y^2+z^2 = 2$ inside $x = \sqrt{y^2+z^2}$, and $\hat{\mathbf{n}}$ is the unit normal with positive x-component.

Answers: **1.** $8\pi a^5/15$ **2.** $7\pi/6$ **3.** -8π **4.** $-1/6$ **5.** 12π **6.** $1915\pi/12$
7. $(64\sqrt{2} - 41)\pi/60$