MATH 3132 Tutorial 6

1. Consider the differential equation

$$(3-x^2)y''+6y=0.$$

(a) Determine a minimm value for the radius of convergence of a power series solution of the differential equation in powers of x.

(b) If the power series $y=\sum_{n=0}^{\infty} c_n x^n$ is substituted into the differential equation, the following recurrence relation is obtained

$$c_{n+2}=\frac{n-3}{3(n+1)}c_n, \quad n\geq 0.$$

Do **NOT** prove this. Use this relation to obtain a general solution for the differential equation. Express any infinite series in sigma notation simplified as much as possible.

2. Consider the differential equation

$$(3-2x^2)y''+4y=0.$$

Substitute $y=\sum_{n=0}^{\infty} a_n x^n$ into the differential equation to obtain a recurrence relation for the coefficients a_n. Do **NOT** find the power series, only the recurrence relation.

3. (a) Show that $x=0$ is an ordinary point of the differential equation

$$(x^2-1)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-5y=0.$$

(b) Suppose a solution of the differential equation in part (a) is represented in terms of its Maclaurin series

$$y=\sum_{n=0}^{\infty} c_n x^n.$$

Find a recurrence relation for the coefficients c_n, and simplify it as much as possible. Do **NOT** attempt to iterate the relation.

4. (a) Determine a guaranteed open interval of convergence for the solution of the differential equation

$$(x^2+3)y''+2xy'-2y=0$$

when the solution is represented in terms of its Maclaurin series $y=\sum_{n=0}^{\infty} c_n x^n$.

(b) Coefficients c_n must satisfy the recurrence relation

$$c_{n+2}=\frac{-(n-1)}{3(n+1)}c_n, \quad n\geq 0.$$

Use this recurrence relation to find a general solution of the differential equation. Write any infinite series in sigma notation, simplified as much as possible.

5. Consider the differential equation

$$y'' + x^2y' + 2xy = 0.$$

(a) What are the ordinary points of the differential equation?

(b) Suppose a solution of the differential equation is represented in terms of its Maclaurin series $y = \sum_{n=0}^{\infty} c_n x^n$. Find a recurrence relation for the c_n, simplified as much as possible. Do **NOT** attempt to iterate the formula.

(c) From your work in part (b), or otherwise, what can you say about c_0, c_1, and c_2?

6. Consider the differential equation

$$y'' - xy' + 2y = 0.$$

(a) Suppose a solution of the differential equation is represented in terms of its Maclaurin series $y = \sum_{n=0}^{\infty} c_n x^n$. Find a recurrence relation for the c_n, simplified as much as possible. Do **NOT** iterate the relation.

(b) From your recurrence relation in part (a), can you say that any of the coefficients c_n must be zero? If so, which ones, and why?

7. When a solution of the differential equation

$$(1 + x^2)y'' + 2xy' - 2y = 0$$

is represented in terms of its Maclaurin series $y = \sum_{n=0}^{\infty} c_n x^n$, coefficients c_n satisfy the recurrence relation

$$(n + 2)(n + 1)c_{n+2} = (2 - n - n^2)c_n, \quad n \geq 0.$$

Do **NOT** show this; assume it.

(a) Using only properties of the differential equation, determine a guaranteed open interval of convergence for the Maclaurin series solution. Explain your reasoning.

(b) Use the recurrence relation to find a general solution of the differential equation. Write any series using sigma notation simplified as much as possible.

Answers: **1.**(a) $\sqrt{3}$ (b) $c_0 \sum_{n=0}^{\infty} \frac{1}{3^{n-1}(2n-1)(2n-3)} x^{2n} + c_1 \left(1 - \frac{x^3}{3}\right)$

2. $a_{n+2} = \frac{2(n-2)}{3(n+2)} a_n$, $n \geq 0$ **3.** $c_{n+2} = \frac{n-5}{n+2} c_n$, $n \geq 0$

4.(a) $-\sqrt{3} < x < \sqrt{3}$ (b) $c_1 x + c_0 \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^n(2n-1)} x^{2n}$

5.(a) All points (b) $c_2 = 0$, $c_{n+3} = \frac{-c_n}{n+3}$, $n \geq 0$ (c) c_0 and c_1 arbitrary

6. (a) $c_{n+2} = \frac{n-2}{(n+1)(n+2)} c_n$, $n \geq 0$ (b) $c_{2n} = 0$ for $n \geq 2$

7.(a) $-1 < x < 1$ (b) $c_1 x + c_0 \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n-1} x^{2n}$