MATH 3132 Tutorial 7

1. (a) Show that the indicial roots for the Frobenius solution $\sum_{n=0}^{\infty} a_n x^{n+r}$ of the differential equation

$$xy'' + 2y' - 3y = 0$$

differ by an integer.

(b) Find the solution of the differential equation corresponding to the smaller indicial root. Express your solution in sigma notation simplified as much as possible. Is it a general solution? What is the radius of convergence of the series?

2. (a) Prove that $x = 0$ is a regular singular point for the differential equation

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 4y = 0.$$

(b) Show that if one assumes a Frobenius solution about $x = 0$, the indicial roots are $r = 1$ and $r = -4$.

(c) Show that a general solution is obtained from the smaller indicial root, but it is not an infinite series.

3. (a) Find all singular points for the differential equation

$$\sin 2x \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 3y = 0.$$

(b) Pick any one singular point from part (a) (your choice which one), and classify it as regular or irregular singular. Justify all statements.

4. (a) Suppose that a Frobenius solution $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$ is assumed for the differential equation

$$(x^2 - x)\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = 0.$$

Find values for the indicial roots.

(b) Using the larger value for r in part (a), find a recurrence relation for the coefficients a_n. Do **NOT** iterate this relation.

(c) If you were to iterate the recurrence relation in part (b), (but don't do it), would you expect to get a general solution of the differential equation? Explain.

5. (a) When a Frobenius solution $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$ is substituted into the differential equation

$$2x(x+1)\frac{d^2y}{dx^2} + 3(x+1)\frac{dy}{dx} - y = 0,$$

the indicial roots are $r = 0$ and $r = -1/2$. The recurrence relation corresponding to $r = 0$ is

$$a_{n+1} = -\frac{2n-1}{2n+3}a_n, \qquad n \geq 0.$$

Iterate this relation to determine coefficients a_n. Write the corresponding solution of the differential equation in sigma notation, simplifed as much as possible.

(b) Is your solution in part (a) a general solution. Would you have expected it to be? Explain.

6. (a) Determine, with justification, whether $x = 0$ is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$4xy'' + 2(1+x)y' + y = 0.$$

(b) If a Frobenius solution $\sum_{n=0}^{\infty} a_n x^{n+r}$ is assumed for the differential equation, find the indicial roots.

(c) For the smaller indicial root, find the recurrence reoation for the coefficients a_n. Do **NOT** iterate it.

Answers: **1.** $r = 0, -1$; $a_1 \sum_{n=0}^{\infty} \frac{3^n}{n!(n+1)!} x^n$; No; $R = \infty$

2. $a_0 x^{-4} + a_5 x$ **3.**(a) $x = n\pi/2$ (b) $x = 0$ is regular singular

4.(a) $r = 0, 4$ (b) $a_{n+1} = \frac{n+2}{n+1} a_n$, $n \geq 0$ (c) No

5.(a) $a_0 \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n-1)(2n+1)} x^n$ (b) No

6.(a) Regular singular (b) $= 0, 1/2$ (c) $a_{n+1} = \frac{-a_n}{2(n+1)}$, $n \geq 0$