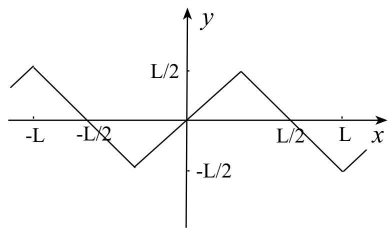
MATH 3132 Tutorial 8

1. Find the Fourier series for the function

$$f(x) = \begin{cases} x - 3, & -3 \le x \le 0, \\ x + 3, & 0 < x < 3. \end{cases} \qquad f(x+6) = f(x).$$

Simplify coefficients as much as possible. Draw a graph on the interval $-6 \le x \le 6$ of the function to which the Fourier series converges.

2. Find the Fourier series of the function in the figure below. Simplify the series as much as possible.



3. Find coefficients c_n so that the series $\sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{4}$ converges to $2x - 1$ on the interval $0 < x < 4$.

4. Suppose the function

$$f(x) = \begin{cases} x, & 0 < x \le 2, \\ 5 - x, & 2 < x < 4. \end{cases} \qquad f(x+4) = f(x)$$

is expanded in a Fourier series. Find the coefficients a_n in the Fourier series, simplified as much as possible. Do **NOT** calculate the coefficients b_n.

5. (a) Explain how, and why, you can expand the function

$$f(x) = \begin{cases} 1, & 0 < x < L/2, \\ -1, & L/2 < x < L. \end{cases}$$

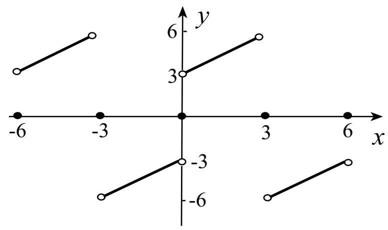
in a series of the form $\sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$.

(b) Find expressions for the coefficients, simplifies ad much as possible.

(c) Use your series to evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$.

6. Calculate, simplified as much as possible, the Fourier sine series of $f(x) = 3 + x$ on the interval $0 \le x \le 3$.

Answers: **1.** $\frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1 + 2(-1)^{n+1}}{n} \sin \frac{n\pi x}{3}$



2. $\dfrac{4L}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{(2n-1)^2}\sin\dfrac{(2n-1)\pi x}{L}$ **3.** $-\dfrac{2}{n\pi}[7(-1)^n+1]$

4. $a_0=3,\ a_n=\dfrac{-4}{n^2\pi^2}[1+(-1)^{n+1}],\ n\geq 1$

5.(b) $\dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{2n-1}\cos\dfrac{(2n-1)\pi x}{L}$ (b) $\pi/4$ **6.** $\dfrac{6}{\pi}\sum_{n=1}^{\infty}\dfrac{1+2(-1)^{n+1}}{n}\sin\dfrac{n\pi x}{3}$