

MATH 3132 Tutorial 9

1. (a) Expand the function $f(x) = 3x$, $0 < x < 2$ in terms of the eigenfunctions of the Sturm-Liouville system

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = 0, \quad y'(2) = 0.$$

- (b) Does the series in part (a) converge to $f(x)$ at $x = 0$ and $x = 2$? You may use the fact that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

2. Determine the value of the constant a so that the functions $f(x) = x$ and $g(x) = x^2 + a$ are orthogonal on the interval $0 \leq x \leq 2$ with respect to the weight function $w(x) = x^2$.
3. (a) Find eigenvalues and eigenfunctions of the Sturm-Liouville system

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4\lambda y = 0, \quad 0 < x < 1, \quad y(0) = 0, \quad y(1) = 0.$$

You may assume that $\lambda \geq 16$.

- (b) What is the weight function for the system?

4. (a) Show that eigenvalues of the Sturm-Liouville system

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \lambda y = 0, \quad 0 < x < 1, \quad y(0) = 0, \quad y(1) = 0$$

are $\lambda_n = 1 + n^2\pi^2$, where $n > 0$ is an integer. You may assume in your calculations that $\lambda > 1$.

- (b) What are corresponding eigenfunctions?

- 5 For what value(s) of constant k are the functions $f(x) = kx^2 + 1$ and $g(x) = 2x$ orthogonal on the interval $0 \leq x \leq 1$ with respect to the weight function $w(x) = x + 1$?

Answers: 1.(a) $\frac{48}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin \frac{(2n-1)\pi x}{4}$ (b) Yes, Yes 2. $-8/3$

3.(a) $1/16 + n^2\pi^2/4$, $y_n(x) = e^{-x/2} \sin(n\pi x)$ (b) $4e^x$

4.(a) $1 + n^2\pi^2$ (b) $e^{-x} \sin(n\pi x)$ 5. $-50/27$