Sturm-Liouville Systems Associated With the Differential Equation

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 < x < L$$

Boundary Conditions	Eigenvalues	Eigenfunctions
$y(0) = 0 = y(L)$	$\frac{n^2\pi^2}{L^2}, \quad n \geq 1$	$y_n(x) = \sin \frac{n\pi x}{L}$
$y'(0) = 0 = y'(L)$	$\frac{n^2\pi^2}{L^2}, \quad n \geq 0$	$y_0(x) = 1, \quad y_n(x) = \cos \frac{n\pi x}{L}$
$y(0) = 0 = y'(L)$	$\frac{(2n-1)^2\pi^2}{4L^2}, \quad n \geq 1$	$y_n(x) = \sin \frac{(2n-1)\pi x}{2L}$
$y'(0) = 0 = y(L)$	$\frac{(2n-1)^2\pi^2}{4L^2}, \quad n \geq 1$	$y_n(x) = \cos \frac{(2n-1)\pi x}{2L}$

Theorem Let p, q, r, r', and $(pr)''$ be real and continuous functions of x for $a \leq x \leq b$, and let $p > 0$ and $r > 0$ for $a \leq x \leq b$. Let l_1, l_2, h_1, and h_2 be real constants independent of λ. Then Sturm-Liouville system 19.1 has an infinity of eigenvalues $\lambda_1 < \lambda_2 < \lambda_3 < \cdots$ (all real), not more than a finite number of which are negative, and $\lim_{n\to\infty} \lambda_n = \infty$. If $f(x)$ is piecewise smooth on $a \leq x \leq b$, then for any x in $a < x < b$,

$$\frac{f(x+) + f(x-)}{2} = \sum_{n=1}^{\infty} c_n y_n(x),$$

where

$$c_n = \frac{1}{F_n} \int_a^b p(x) f(x) y_n(x)\, dx \qquad \text{and} \qquad F_n = \int_a^b p(x)[y_n(x)]^2 dx.$$