Sample Test 2 MATH3132

Time: 90 Minutes

1. Evaluate

$$\iint_S [(2x + y + 3z)\hat{\mathbf{i}} - x\hat{\mathbf{j}} - z\hat{\mathbf{k}}] \cdot \hat{\mathbf{n}}\, dS$$

where S is that part of the surface $x + 2y + 3z = 6$ in the first octant and $\hat{\mathbf{n}}$ is the unit downward normal to S.

Answer: -3

2. Evaluate the surface integral

$$\oiint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS,$$

where $\mathbf{F} = xy^2\hat{\mathbf{i}} + x^2y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, S is the surface that encloses the region bounded by $x^2 + y^2 = 4$, $z = 1$, and $z = 4$, and $\hat{\mathbf{n}}$ is the unit outer normal to S.
Answer: 36π

3. Use Stokes's theorem to evaluate the line integral

$$\oint_C y^3 z\, dx - x^3 z\, dy + 4\, dz,$$

where C is the curve of intersection of the paraboloid $z = 2 + x^2 + y^2$ and the plane $z = 5$, directed clockwise as viewed from the point $(0, 0, 7)$.

Answer:$135\pi/2$

4. (a) Find all singular points for the differential equation

$$xy'' + 2y' + y = 0.$$

(b) Can you predict a minimum value for the radius of convergence for the Maclaurin series solution of the differential equation.

(c) Find the Maclaurin series solution of the differential equation. Express your answer in sigma notation simplified as much as possible. What is the radius of convergence of the series?

Answer: (a) $x = 0$ (b) No (c) $a_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} x^n$, $R = \infty$