Sample Test 1 MATH3132

Time: 75 Minutes

1. Evaluate the line integral

$$\int_C xy\,ds$$

where C is that part of the curve $x^2+y^2=4$, $x+z=4$ in the first octant from $(2,0,2)$ to $(0,2,4)$.

Answer: $(16\sqrt{2}-8)/3$

2. Evaluate the line integral

$$\int_C -\frac{y}{x^2}dx + \left(\frac{1}{x}+z\right)dy + (y-1)dz$$

where C is the curve $x=z^2+1$, $y=z$ from $(1,0,0)$ to $(10,3,3)$.

Answer: $63/10$

3. Find a value for the constant c in order that the vector field

$$\mathbf{F}(x,y,z) = (cx^2+y)\hat{\mathbf{i}} + x^2z\hat{\mathbf{j}} + xyz^2\hat{\mathbf{k}}$$

have zero divergence at the point $(1,-2,3)$.

Answer: $c=6$

4. Evaluate the line integral

$$\oint_C (3x^2-y^2+3ye^{3x})dx + (2x^2+1+e^{3x})dy$$

where C is that curve bounding the area enclosed by $y=4-x^2$, $y=0$.

Answer: $-512/15$

5. Evaluate

$$\iint_S (x^2+y^2)dS$$

where S is that part of the surface $2x+y-z=1$ inside $x^2+y^2=2$.

Answer: $2\sqrt{6}\pi$

6. You are to evaluate the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

once around the curve $x^2 + y^2 + z^2 = 3$, $2z = x^2 + y^2$ directed counterclockwise as viewed from the origin. You are told that

$$\nabla \times \mathbf{F} = (x^2 + y^2)\hat{\mathbf{i}} - xz^2\hat{\mathbf{j}} + z^2 e^{x^2}\hat{\mathbf{k}}.$$

Set up, but do not evaluate, a double iterated integral in polar coordinates that has the same value as the line integral.

Answer: $-\displaystyle\int_0^{2\pi}\int_0^{\sqrt{2}} e^{r^2\cos^2\theta} r\,dr\,d\theta$