

MATH3132 Sample Test 2 60 minutes

1. (a) Find the Fourier series for the function

$$f(x) = 3|x|, \quad -2 \leq x \leq 2, \quad f(x+4) = f(x).$$

Simplify the series as much as possible.

- (b) Use the Fourier series in part (a) to find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

Answer: (a) $3 - \frac{24}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{2}$ (b) $\pi^2/8$

2. Let $f(x) = x^2$, and $g(x) = 2x - 3$.

- (a) Determine whether the functions are orthogonal on the interval $0 \leq x \leq 2$ with respect to the weight function $w(x) = 1$.
- (b) Determine whether the functions are orthogonal on the interval $0 \leq x \leq 2$ with respect to the weight function $w(x) = x$.

Answer: (a) Yes (b) No

3. Suppose a Maclaurin series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is to be found for the differential equation

$$y'' - xy' + 2y = 0.$$

- (a) Find a recurrence relation for the coefficients a_n , simplified as much as possible. Do **NOT** attempt to iterate it.
- (b) Can you predict which, if any, coefficients will be equal to zero?

Answer: (a) $a_{n+2} = \frac{(n-2)a_n}{(n+1)(n+2)}$ (b) $0 = c_4 = c_6 = c_8 = \dots$