MATH3132 Sample Test 2 60 minutes

1. (a) Show that the indicial roots for the Frobenius solution $\sum_{n=0}^{\infty} a_n x^{n+r}$ for the differential equation

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$$

differ by an integer.

(b) Find the solution of the differential equation corresponding to the smaller indicial root. Express your answer in sigma notation simplified as much as possible. Is it a general solution? What is the radius of convergence of the series?

Answer:(a) $r = 0, -1$ (b) $a_1 \sum_{n=0}^{\infty} \frac{3^n}{n!(n+1)!} x^n$ Not general $R = \infty$

2. Find the Fourier series for the function

$$f(x) = \begin{cases} |x| + 2, & -2 \le x \le 2, \\ 0 & 2 \le x \le 6, \end{cases} \qquad f(x+8) = f(x).$$

(b) On the interval $-4 \le x \le 8$, Draw a graph of the function to which the series converges.

Answer: (a) $\frac{3}{2} + \sum_{n=1}^{\infty} \left\{ \frac{8}{n^2\pi^2} \left[\cos\frac{n\pi}{2} - 1 \right] + \frac{8}{n\pi} \sin\frac{n\pi}{2} \right\} \cos\frac{n\pi x}{4}$