

MECH 3460 Heat Transfer: Fin Equation Summary

Fins of Uniform Cross-Sectional Area

Case	Tip Condition	Temperature Distribution $\frac{\theta(x)}{\theta_b}$	Fin Heat Rate, q_f [W]
A	Convection	$\frac{\left[\cosh(m(L-x)) + \left(\frac{h}{m k} \right) \sinh(m(L-x)) \right]}{\left[\cosh(mL) + \left(\frac{h}{m k} \right) \sinh(mL) \right]}$	$M \frac{\left[\sinh(mL) + \left(\frac{h}{m k} \right) \cosh(mL) \right]}{\left[\cosh(mL) + \left(\frac{h}{m k} \right) \sinh(mL) \right]}$
B	Adiabatic	$\frac{\cosh(m(L-x))}{\cosh(mL)}$	$M \tanh(mL)$
C	Temperature Specified	$\frac{\left[\left(\frac{\theta_L}{\theta_b} \right) \sinh(mx) + \sinh(m(L-x)) \right]}{\sinh(mL)}$	$M \frac{\left[\cosh(mL) - \left(\frac{\theta_L}{\theta_b} \right) \right]}{\sinh(mL)}$
D	Infinite Length	e^{-mx}	M

$$\begin{aligned}\theta(x) &= (T(x) - T_\infty) \\ \theta_b &= (T_b - T_\infty) = (T(0) - T_\infty) \\ \theta_L &= (T_L - T_\infty) = (T(L) - T_\infty) \\ m &= \left[\frac{h P}{k A_c} \right]^{1/2} \\ M &= \theta_b [h P k A_c]^{1/2}\end{aligned}$$

$$\begin{aligned}P &\equiv \text{perimeter} \\ A_c &\equiv \text{cross-sectional area} \\ k &\equiv \text{thermal conductivity} \\ h &\equiv \text{convection heat transfer coefficient} \\ T_b &\equiv \text{fin base temperature} \\ T_\infty &\equiv \text{ambient fluid temperature} \\ L &\equiv \text{fin length}\end{aligned}$$

$$\begin{aligned}\sinh(x) &= \frac{1}{2} (e^x - e^{-x}) \\ \cosh(x) &= \frac{1}{2} (e^x + e^{-x}) \\ \tanh(x) &= \frac{\sinh(x)}{\cosh(x)}\end{aligned}$$

