4.1

$$2.0\,\phi_1 - \phi_2 = 10.0$$

$$-\phi_1 + 6.0\,\phi_2 = 6.0$$

(a) Simultaneous solution

• Equation substitution

From the first equation $\phi_2 = 2.0\,\phi_1 - 10.0$

Substitute this into second equation

$$-\phi_1 + 6.0\,(2.0\,\phi_1 - 10.0) = 6.0$$

$$\Rightarrow \quad -\phi_1 + 12.0\,\phi_1 - 60.0 = 6.0$$

$$\Rightarrow \quad 11.0\,\phi_1 = 66.0 \quad \Rightarrow \quad \phi_1 = \frac{66.0}{11.0} = 6.0$$

From previous equation for ϕ_2

$$\phi_2 = 2.0\,(6.0) - 10.0 = 12.0 - 10.0 = 2.0$$

Solution: $\boxed{\begin{array}{l}\phi_1 = 6.0 \\ \phi_2 = 2.0\end{array}}$

CONTINUED

4.1 (Continued

Alternative: Cramer's Rule $A = \begin{bmatrix} 2.0 & -1.0 \\ -1.0 & 6.0 \end{bmatrix}$

$\det A = 12 - 1 = 11$

$\phi_1 = \frac{1}{11} \det \begin{bmatrix} 10.0 & -1.0 \\ 6.0 & 6.0 \end{bmatrix} = \frac{1}{11}(60 + 6) = \frac{66}{11} = 6.0$ ✓

$\phi_2 = \frac{1}{11} \det \begin{bmatrix} 2.0 & 10.0 \\ -1.0 & 6.0 \end{bmatrix} = \frac{1}{11}(12 + 10) = \frac{22}{11} = 2.0$ ✓

1(b) Iteration from $\phi_1 = \phi_2 = 0.0$ initial guess (use most recent values as they become available)

Iteration	ϕ_1	ϕ_2
0	0.0	0.0
1	5.0	1.8333
2	5.9167	1.9861
3	5.9931	1.9988
4	5.9994	1.9999

$$3\phi_1 - \phi_2 = 14 \qquad (1)$$

$$-\phi_1 + 2\phi_2 = -3 \qquad (2)$$

(a) Simultaneous Solution: (by substitution)

From Eqn (2)

$$\phi_1 = 2\phi_2 + 3 \qquad (3)$$

Substitute Eqn (3) into Eqn (1)

$$3(2\phi_2 + 3) - \phi_2 = 14$$

$$6\phi_2 - \phi_2 + 9 = 14$$

$$5\phi_2 = 5 \Rightarrow \phi_2 = 1$$

From Eqn. (3) $\phi_1 = 2(1) + 3 = 5$

Solution $\boxed{\phi_1 = 5 \qquad \phi_2 = 1}$

(b) Gauss-Seidel Iteration Summary

Iteration	ϕ_1	ϕ_2
0	0	0
1	4.66667	0.83334
2	4.94445	0.97223
3	4.99074	0.99537
4	4.99846	0.99923

4.3

$$2\Phi_1 - \Phi_2 = 14$$
$$-3\Phi_1 + 4\Phi_2 = -46$$

(a) Gauss - Seidel Iteration

$$\Phi_1 = \frac{14+\Phi_2}{2} = 0.5\Phi_2 + 7.0$$

$$\Phi_2 = \frac{-46+3\Phi_1}{4} = 0.75\Phi_1 - 11.5$$

Iteration	Φ_1	Φ_2
0	2.0	2.0
1	8.0	-5.5
2	4.25	-8.3125
3	2.84375	-9.36719
4	2.31641	-9.76270
5	2.11865	-9.91101

(b)

$$\left|\frac{\left(\Phi_1^{k=5} - \Phi_1^{k=4}\right)}{\Phi_1^{k=5}}\right| \times 100 = 9.33\%$$

$$\left|\frac{\left(\Phi_2^{k=5} - \Phi_2^{k=4}\right)}{\Phi_2^{k=5}}\right| \times 100 = 1.50\%$$

(c) Substitute equation for Φ_2 into Φ_1

$$\Phi_1 = 0.5\{0.75\Phi_1 - 11.5\} + 7.0 = 0.375\Phi_1 + 1.25$$

$$\Rightarrow \quad \Phi_1 = \frac{1.25}{0.625} = \underline{\underline{2.0}}$$

$$\Rightarrow \quad \Phi_2 = 0.75(2.0) - 11.5 = \underline{\underline{-10.0}}$$

$$\boxed{\begin{aligned}\Phi_1 &= 2.0 \\ \Phi_2 &= -10.0\end{aligned}}$$

$$2\phi_1 + \phi_2 = 17 \qquad (1)$$

$$3.5\phi_1 + 2\phi_2 = 28 \qquad (2)$$

(a) Graphical solution

Eqn (1) $\Rightarrow$ $\phi_2 = 17 - 2\phi_1$ (3)

ϕ_2 intercept is 17
ϕ_1 intercept is 8.5

Eqn (2) $\Rightarrow$ $\phi_2 = 14 - 1.75\phi_1$ (4)

ϕ_2 intercept is 14
ϕ_1 intercept is 8

See graph on the next page.

Solution from the graph is $\boxed{\begin{array}{l}\phi_1 = 12 \\ \phi_2 = -7\end{array}}$

(b) From Eqn (1) $\phi_1 = 8.5 - 0.5\phi_2$ (5)

Substitute Eqn (5) into Eqn (2)

$$3.5\left[8.5 - 0.5\phi_2\right] + 2\phi_2 = 28$$

$$29.75 - 1.75\phi_2 + 2\phi_2 = 28$$

$$0.25\phi_2 = -1.75 \Rightarrow \boxed{\phi_2 = -7}$$

From Eqn (5) $\phi_1 = 8.5 - 0.5(-7) \Rightarrow \boxed{\phi_1 = 12}$

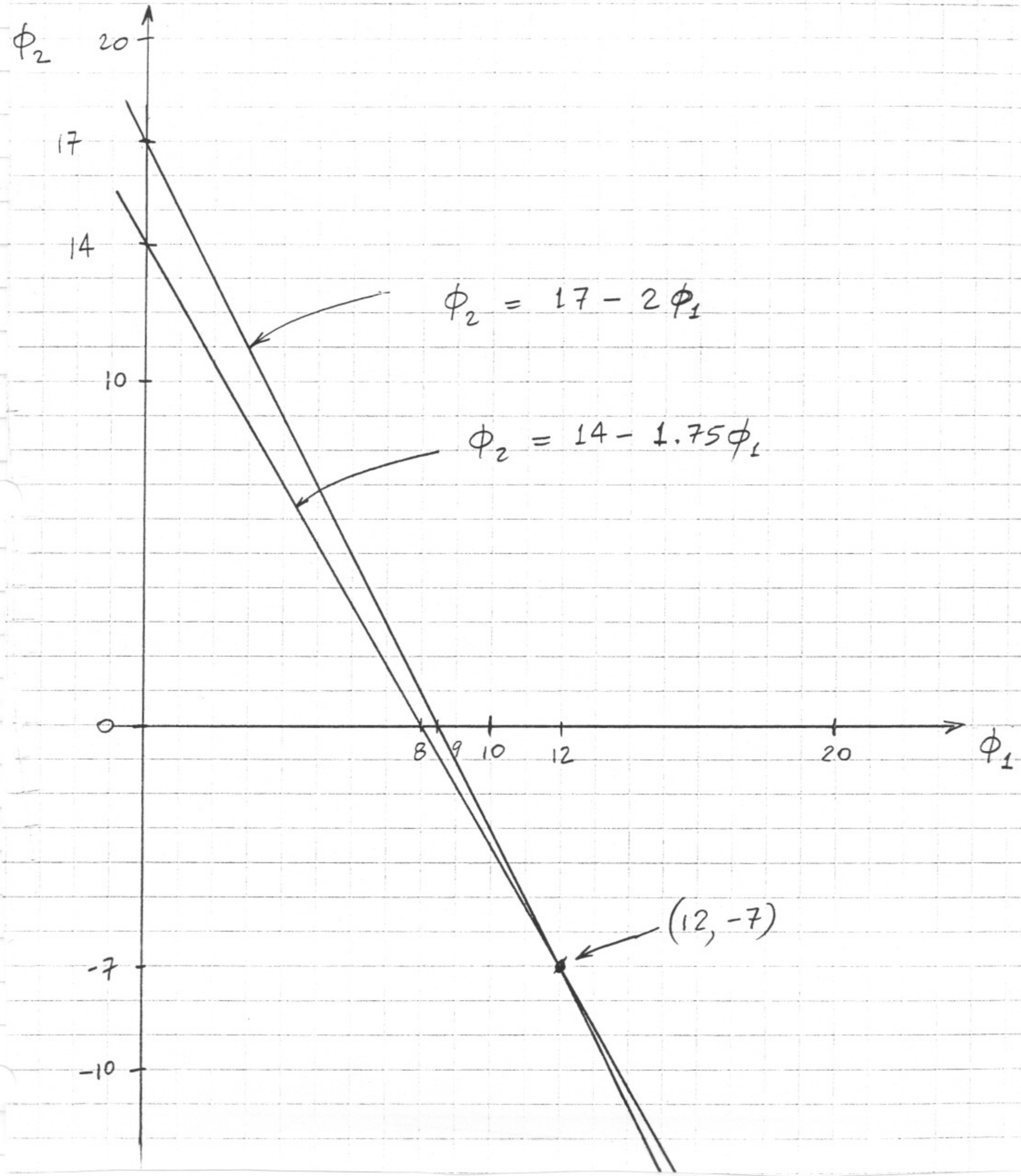
(c) Check answers

Eqn (1) LHS $= 2(12) + (-7) = 24 - 7 = 17$ = RHS ✓

Eqn (2) LHS $= 3.5(12) + 2(-7) = 42 - 14 = 28$ = RHS ✓

CONTINUED

#2 (a) graph


ϕ_2
20
17
14
10
0
-7
-10
8
9
10
12
20
ϕ_1
$\phi_2 = 17 - 2\phi_1$
$\phi_2 = 14 - 1.75\phi_1$
(12, -7)

$$x_1 - 3x_2 + 12x_3 = 269 \quad (1)$$

$$5x_1 - 13x_2 + 2x_3 = 379 \quad (2)$$

$$x_1 - 14x_2 = -45 \quad (3)$$

(a) Gauss Elimination

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 5 & -13 & 2 & 379 \\ 1 & -14 & 0 & -45 \end{array}\right|$$

row 2 − 5 times row 1 ; row 3 − row 1

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 0 & 2 & -58 & -966 \\ 0 & -11 & -12 & -314 \end{array}\right|$$

row 3 + $\frac{11}{2}$ · row 2

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 0 & 2 & -58 & -966 \\ 0 & 0 & -331 & -5627 \end{array}\right|$$

$$\Rightarrow \quad x_3 = \frac{-5627}{-331} = 17 \leftarrow$$

$$x_2 = \frac{-966 + 58(17)}{2} = 10 \leftarrow$$

$$x_1 = \frac{269 - 12(17) + 3(10)}{1} = 95 \leftarrow$$

CONTINUED

(b) Gauss Jordan 5

Start with last step of Gauss Elimination

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 0 & 2 & -58 & -966 \\ 0 & 0 & -331 & -5627 \end{array}\right|$$

row 2 ÷ 2; row 3 ÷ -331

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 0 & 1 & -29 & -483 \\ 0 & 0 & 1 & 17 \end{array}\right|$$

row 2 + 29 times row 3

$$\left|\begin{array}{ccc|c} 1 & -3 & 12 & 269 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 17 \end{array}\right|$$

row 1 + 3 times row 2

$$\left|\begin{array}{ccc|c} 1 & 0 & 12 & 299 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 17 \end{array}\right|$$

CONTINUED

row 1 − 12 time row 3

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 95 \\ 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 17 \end{array}\right| \Bigg\}$$

Solution
$x_1 = 95$
$x_2 = 10$
$x_3 = 17$

(c) Gauss Seidel iteration, $\varepsilon_s = 1\%$
$x_1 = x_2 = x_3 = 0$ initial guess.

Notes 1. The equations cannot be used in the given order for x_1, x_2, & x_3 respectively because Eqn (3) cannot be used as an equation for x_3.

2. Use diagonal dominance as a criteria for re-arranging the equations.

→ • Use Eqn (3) for x_2
• Use Eqn (1) for x_3
• Use Eqn (2) for x_1

CONTINUED

4.5 (Continued)

$$\Rightarrow \quad x_1 = \frac{379 + 13x_2 - 2x_3}{5} \qquad (4)$$

$$x_2 = \frac{+45 + x_1}{14} \qquad (5)$$

$$x_3 = \frac{269 - x_1 + 3x_2}{12} \qquad (6)$$

Iteration 1

$$x_1 = \frac{379 + 13(0) - 2(0)}{5} = 75.800$$

$$x_2 = \frac{45 + 75.80}{14} = 8.6286$$

$$x_3 = \frac{269 - 75.8 + 3(8.6286)}{12} = 18.2572$$

CONTINUED

4.5 (continued)

Iteration 2

$$x_1 = \frac{379 + 13(8.6286) - 2(18.2572)}{5} = 90.931$$

$$(\varepsilon_{a_1} = 16.6\%)$$

$$x_2 = \frac{45 + 90.931}{14} = 9.7094$$

$$(\varepsilon_{a_2} = 11.1\%)$$

$$x_3 = \frac{269 - 90.931 + 3(9.7094)}{12} = 17.266$$

$$(\varepsilon_{a_3} = -5.7\%)$$

Iteration 3

$$x_1 = \frac{379 + 13(9.7094) - 2(17.266)}{5} = 94.138$$

$$(\varepsilon_{a_1} = 3.4\%)$$

$$x_2 = \frac{45 + 94.138}{14} = 9.9384$$

$$(\varepsilon_{a_2} = 2.3\%)$$

$$x_3 = \frac{269 - 94.138 + 3(9.9384)}{12} = 17.056$$

$$(\varepsilon_{a_3} = -1.2\%)$$

CONTINUED

Iteration 4

$$x_1 = \frac{379 + 13(9.9384) - 2(17.056)}{5} = 94.817$$

$(\varepsilon_{a_1} = 0.72\,\%)$

$$x_2 = \frac{45 + 94.817}{14} = 9.9869$$

$(\varepsilon_{a_2} = 0.49\,\%)$

$$x_3 = \frac{269 - 94.817 + 3(9.9869)}{12} = 17.012$$

$(\varepsilon_{a_3} = -0.26\,\%)$

stop iteration because $|\varepsilon_a|_{max} < 1\,\%$

Solution

$x_1 = 94.817$

$x_2 = 9.9869$

$x_3 = 17.012$

4.6

(a) Gauss Elimination

$$\left[\begin{array}{ccc|c} 1 & -12 & 5 & 16 \\ 3 & 0 & -2 & 272 \\ 1 & -3 & 10 & 146 \end{array}\right]$$

row 2 $- 3 *$ row 1 ; row 3 $-$ row 1

$$\left[\begin{array}{ccc|c} 1 & -12 & 5 & 16 \\ 0 & 36 & -17 & 224 \\ 0 & 9 & 5 & 130 \end{array}\right]$$

4 row 3 $-$ row 2

$$\left[\begin{array}{ccc|c} 1 & -12 & 5 & 16 \\ 0 & 36 & -17 & 224 \\ 0 & 0 & 37 & 296 \end{array}\right]$$

$$x_3 = \frac{296}{37} = 8 \leftarrow$$

$$x_2 = \frac{224 + 17(8)}{36} = 10 \leftarrow$$

$$x_1 = \frac{16 + 12(10) - 5(8)}{1} = 96 \leftarrow$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 96 \\ 10 \\ 8 \end{Bmatrix} \leftarrow$$

CONTINUED

3 (b) check answers

$$96 - 12(10) + 5(8) = 16 \checkmark$$

$$3(96) - 2(8) = 272 \checkmark$$

$$96 - 3(10) + 10(8) = 146 \checkmark$$

(c) Gauss-Seidel

- Cannot use second row equation for x_2

→ Use it for x_1 (because the coefficient on x_1 is largest)

$$\boxed{x_1 = \frac{1}{3}(2x_3 + 272)} \qquad (1)$$

- Likewise, choose the first equation for x_2 and the last equation for x_3:

$$\boxed{x_2 = \frac{1}{12}(x_1 + 5x_3 - 16)} \qquad (2)$$

$$\boxed{x_3 = \frac{1}{10}(-x_1 + 3x_2 + 146)} \qquad (3)$$

CONTINUED

4.6 (continued)

Iterations $x_1^0 = x_2^0 = x_3^0 = 0$

k=1

$$x_1^1 = \frac{(2(0) + 272)}{3} = \underline{90.667}$$

$$x_2^1 = \frac{1}{12}(90.667 + 5(0) - 16) = \underline{6.2223}$$

$$x_3^1 = \frac{1}{10}(-90.667 + 3(6.2223) + 146) = \underline{7.4000}$$

k=2

$$x_1^2 = \frac{1}{3}(2(7.4000) + 272) = \underline{95.600}$$

$$\epsilon_{a_1} = \frac{(95.600 - 90.667)}{95.600} \times 100 = 5.16\%$$

$$x_2^2 = \frac{1}{12}(95.600 + 5(7.4000) - 16) = \underline{9.7167}$$

$$\epsilon_{a_2} = 35.96\%$$

$$x_3^2 = \frac{1}{10}(-95.600 + 3(9.7167) + 146) = \underline{7.9550}$$

$$\epsilon_{a_3} = 6.98\%$$

CONTINUED

4.6 (continued)

$k=3$

$x_1^3 = \frac{1}{3}(2(7.9550 + 272) = \underline{95.970}$

$\epsilon_{a_1} = 0.39\ \%$ ok

$x_2^3 = \frac{1}{12}(95.970 + 5(7.9550) - 16) = \underline{9.9788}$

$\epsilon_{a_2} = 2.63\ \%$ too large $(>1\%)$

$x_3^3 = \frac{1}{10}(-95.970 + 3(9.9788) + 146) = \underline{7.9966}$

$\epsilon_{a_3} = 0.52\ \%$ ok

$k=4$

$x_1^4 = \frac{1}{3}(2(7.9966) + 272) = \boxed{95.998}$

$\epsilon_{a_1} = 0.029\ \%$ ok ✓

$x_2^4 = \frac{1}{12}(95.998 + 5(7.9966) - 16) = \boxed{9.9984}$

$\epsilon_{a_2} = 0.20\ \%$ ok ✓

$x_3^4 = \frac{1}{10}(-95.998 + 3(9.9984) + 146) = \boxed{7.9997}$

$\epsilon_{a_3} = 0.039\ \%$ ok ✓

4.7

(a)
$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -25 \\ 10 \\ -10 \end{bmatrix}$$

Gauss- Elimination

row 2 + $\frac{1}{3}$ row 1

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ 0 & 1.3333 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -2.6667 \\ 10 \\ -10 \end{bmatrix}$$

row 3 + $\frac{3}{4}$ row 2

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ 0 & 1.3333 & -1 & 0 \\ 0 & 0 & 1.25 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -2.6667 \\ 8.0 \\ -10.0 \end{bmatrix}$$

row 4 + $\frac{8}{5}$ * row 3

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ 0 & 1.3333 & -1 & 0 \\ 0 & 0 & 1.25 & -1 \\ 0 & 0 & 0 & 1.4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -2.6667 \\ 8.0 \\ 2.80 \end{bmatrix}$$

CONTINUED

Solve
$$x_4 = \frac{2.80}{1.40} = 2.0$$

$$x_3 = \frac{1}{1.25}(8 + 2.0) = 8.0$$

$$x_2 = \frac{1}{1.3333}(-2.6667 + 8.0) = 4.0$$

$$x_1 = \frac{1}{3}(67 + 2(4.0)) = 25.0$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 25.0 \\ 4.0 \\ 8.0 \\ 2.0 \end{Bmatrix}$$

2(b) Gauss-Seidel

$$x_1^{k+1} = \frac{(67 + 2x_2^{k})}{3} \qquad (1)$$

$$x_2^{k+1} = \frac{(-25 + x_1^{k+1} + x_3^{k})}{2} \qquad (2)$$

$$x_3^{k+1} = \frac{(10 + x_2^{k+1} + x_4^{k})}{2} \qquad (3)$$

$$x_4^{k+1} = \frac{(-10 + 2x_3^{k+1})}{3} \qquad (4)$$

$$x_1^{0} = x_2^{0} = x_3^{0} = x_4^{0}$$

CONTINUED

4.7 (Continued)

Iteration 1

$$x_1^1 = \frac{67 + 2(0)}{3} = 22.333$$

$$x_2^1 = \frac{(-25 + 22.333 + 0)}{2} = -1.3335$$

$$x_3^1 = \frac{(10 + (-1.3335) + 0)}{2} = 4.3333$$

$$x_4^1 = \frac{(-10 + 2(4.3333))}{3} = -0.4445$$

Iteration 2

$$x_1^2 = \frac{67 + 2(-1.3335)}{3} = 21.444$$

$$x_2^2 = \frac{-25 + 21.444 + 4.3333}{2} = 0.38865$$

$$x_3^2 = \frac{10 + 0.38865 + (-0.4445)}{2} = 4.9721$$

$$x_4^2 = \frac{-10 + 2(4.9721)}{3} = -0.0186$$

Summary

Iteration	x_1^k	x_2^k	x_3^k	x_4^k
0	0	0	0	0
1	22.333	-1.3335	4.3333	-0.4445
2	21.444	0.38865	4.9721	-0.0186

4.8

$$\begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}$$

(a) Gauss Elimination

Row 2 − 2 Row 3

Row 3 − $\frac{1}{3}$ Row 1

$$\left[\begin{array}{ccc|c} 3 & 1 & 6 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 2/3 & -1 & 10/3 \end{array}\right]$$

$$\frac{12}{3} - \frac{2}{3} = \frac{10}{3}$$

Row 3 + $\frac{2}{3}$ Row 2

$$\left[\begin{array}{ccc|c} 3 & 1 & 6 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & -\frac{1}{3} & \frac{8}{3} \end{array}\right]$$

$$\frac{10}{3} - \frac{2}{3} = \frac{8}{3}$$

$$x_3 = \frac{\frac{8}{3}}{\left(-\frac{1}{3}\right)} = -8$$

$$x_2 = \frac{-1 - (-8)}{-1} = \frac{-1+8}{-1} = -7$$

$$x_1 = \frac{2 - 6(-8) - (-7)}{3} = \frac{2+48+7}{3} = \frac{57}{3} = 19$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 19 \\ -7 \\ -8 \end{Bmatrix}$$

CONTINUED

4.8 (continued)

(b) Gauss Seidel

$$x_1^{k+1} = \frac{2 - x_2^k - 6x_3^k}{3} \quad (1)$$

$$x_2^{k+1} = \frac{7 - 2x_1^{k+1} - 3x_3^k}{1} \quad (2)$$

$$x_3^{k+1} = \frac{4 - x_1^{k+1} - x_2^{k+1}}{1} \quad (3)$$

$k=0$ $\quad x_1^k = 0 \quad x_2^k = 0 \quad x_3^k = 0$

$k=1$

$$x_1^1 = \frac{2 - 0 - 0}{3} = \frac{2}{3} = 0.66667$$

$$x_2^1 = 7 - 2(0.66667) - 3(0) = 5.6667$$

$$x_3^1 = 4 - 0.66667 - 5.6667 = -2.3333$$

$k=2$

$$x_1^2 = \frac{2 - 5.6667 - 6(-2.3333)}{3} = 3.4444$$

$$x_2^2 = 7 - 2(3.4444) - 3(-2.3333) = 7.1111$$

$$x_3^3 = 4 - 3.4444 - 7.1111 = -6.5555$$

4.9

TDMA solution of the equation set:

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 625 \\ 25 \\ 1225 \end{bmatrix}$$

In terms of "compass point" notation

$a_{p_1} = 3$ $\quad a_{W_1} = 0$ $\quad a_{E_1} = 1$ $\quad b_{p_1} = 625$

$a_{p_2} = 2$ $\quad a_{W_2} = 1$ $\quad a_{E_2} = 1$ $\quad b_{p_2} = 25$

$a_{p_3} = 3$ $\quad a_{W_3} = 1$ $\quad a_{E_3} = 0$ $\quad b_{p_3} = 1225$

(a) $i = 1$

$$P_1 = \frac{a_{E_1}}{a_{p_1}} = \frac{1}{3} = 0.333333$$

$$Q_1 = \frac{b_{p_1}}{a_{p_1}} = \frac{625}{3} = 208.333$$

$i=2$

$$P_2 = \frac{a_{E_2}}{a_{p_2} - a_{W_2} P_1} = \frac{1}{2 - (1)(0.333333)} = 0.600000$$

$$Q_2 = \frac{a_{W_2} Q_1 + b_{p_1}}{a_{p_2} - a_{W_2} P_1} = \frac{(1)(208.333) + 25}{2 - (1)(0.333333)} = 140.000$$

$i=3$

$$P_3 = \frac{a_{E_3}}{a_{p_3} - a_{W_3} P_2} = 0$$

$$Q_3 = \frac{a_{W_3} Q_2 + b_{p_3}}{a_{p_3} - a_{W_3} P_2} = \frac{(1)((140.000) + 1225}{3 - (1)(0.600000)} = 568.75$$

CONTINUED

4.9 (continued)

(b) Back substitution

$$T_3 = Q_3 = 568.75$$

$$T_2 = P_2 T_3 + Q_2 = 0.600000(568.75) + 140.000$$

$$T_2 = 481.25$$

$$T_1 = P_1 T_2 + Q_1 = 0.333333(481.25) + 208.333$$

$$T_1 = 368.75$$

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 368.75 \\ 481.25 \\ 568.75 \end{Bmatrix}$$

(c) Check

$$(3)(368.75) - (1)(481.25 = 625 \checkmark$$

$$(-1)(368.75) + (2)(481.25) - (1)(568.75) = 25 \checkmark$$

$$-(1)(481.25) + (3)(568.75) = 1225 \checkmark$$

4.10

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -25 \\ 10 \\ -10 \end{bmatrix}$$

(a) $a_{W_1} = 0 \quad a_{P_1} = 3 \quad a_{E_1} = 2 \quad b_{P_1} = 67$

$a_{W_2} = 1 \quad a_{P_2} = 2 \quad a_{E_2} = 1 \quad b_{P_2} = -25$

$a_{W_3} = 1 \quad a_{P_3} = 2 \quad a_{E_3} = 1 \quad b_{P_3} = 10$

$a_{W_4} = 2 \quad a_{P_4} = 3 \quad a_{E_4} = 0 \quad b_{P_4} = -10$

$i = 1$

$$P_1 = \frac{a_{E1}}{a_{P1}} = \frac{2}{3} = 0.66667$$

$$Q_1 = \frac{b_{P1}}{a_{P1}} = \frac{67}{3} = 22.333$$

$i = 2$

$$P_2 = \frac{a_{E2}}{(a_{P2} - a_{W2} P_1)} = \frac{1}{[(2 - 1(0.66667)]} = \frac{1}{1.3333} = 0.75$$

$$Q_2 = \frac{a_{W2} Q_1 + b_{P2}}{(a_{P2} - a_{W2} P_1)} = \frac{(1)(22.333) + -25}{1.3333} = -2.0$$

$i = 3$

$$P_3 = \frac{a_{E3}}{a_{P3} - a_{W3} P_2} = \frac{1}{[2 - 1(0.75)]} = \frac{1}{1.25} = 0.80$$

$$Q_3 = \frac{a_{W3} Q_2 + b_{P3}}{a_{P3} - a_{W3} P_2} = \frac{1(-2.0) + 10}{1.25} = \frac{8}{1.25} = 6.40$$

CONTINUED

$i=4$

$$P_4 = 0$$

$$Q_4 = \frac{a_{W4} Q_3 + b_{P4}}{a_{P4} - a_{W4} P_3} = \frac{(2)(6.40) + (-10)}{[3 - (2)(0.8)]} = \frac{2.8}{1.4} = 2.0$$

(b)

$$\phi_4 = Q_4 = 2$$

$$\phi_3 = P_3 \phi_4 + Q_3 = (0.8)(2) + 6.40 = 8.0$$

$$\phi_2 = P_2 \phi_3 + Q_2 = (0.75)(8.0) - 2.0 = 4.0$$

$$\phi_1 = P_1 \phi_2 + Q_1 = (0.66667)(4.0) + 22.333 = 25.0$$

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{Bmatrix} = \begin{Bmatrix} 25.0 \\ 4.0 \\ 8.0 \\ 2.0 \end{Bmatrix}$$

(c) Substitution Check

row1 $3(25.0) - 2(4.0) = 75 - 8 = 67$ ✓

row2 $-25 + 2(4.0) - 8.0 = -25 + 8 - 8 = -25$ ✓

row3 $-4.0 + 2(8.0) - 2.0 = -4 + 16 - 2 = 10$ ✓

row4 $-2(8.0) + 3(2.0) = -16 + 6 = -10$ ✓