5.1

Computer code results $T_1 = 320.46$ [K]
$T_2 = 358.19$ [K]
$T_3 = 392.72$ [K]

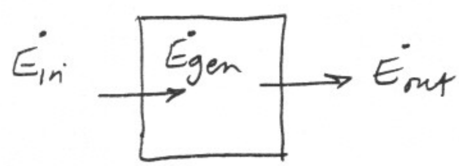
$$\Delta x = \frac{L}{3} = \frac{0.6\,[m]}{3} = 0.2\,[m]$$

$k = 100 \left[\frac{W}{mK}\right]$ $\qquad \dot{Q}''' = 8000\,[W/m^3]$

$A = 1\,[m^2]$

Left B.C $\quad T_L = 300\,[K]$
Right B.C. $\quad T_\infty = 800\,[K]$
$h_\infty = 40\,[W/m^2K]$

Energy Conservation on a volume



$$\boxed{\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}}$$

(a) Energy conservation for C.V. for T_1

$$\dot{E}_{in} = \frac{-k(T_1 - T_L)}{\left(\frac{\Delta x}{2}\right)} = -100\left[\frac{W}{mK}\right]\frac{1}{\left(\frac{0.2}{2}\right)(m)}(320.46 - 300)\,[K] = -20,460\,[W]$$

$$\dot{E}_{gen} = \dot{Q}'''(Vol) = 8000\left[\frac{W}{m^3}\right] 1\,[m^2]\,0.2\,[m] = 1600\,[W]$$

$$\dot{E}_{out} = -\frac{k}{\Delta x}(T_2 - T_1) = \frac{-100}{0.2}(358.19 - 320.46) = -18,865\,[W]$$

Balance? $\quad -20,460 + 1600 = -18,860 \simeq -18,865$
(balances within 0.001 %)

CONTINUED

C.V. for T_2 $\quad \dot{E}_{in} = -18,865\ [W]$

$$\dot{E}_{gen} = 1600\ [W]$$

$$\dot{E}_{out} = -\frac{100}{0.2}(392.72 - 358.19) = -17,265\ [W]$$

Balance? $\quad -18,865 + 1600 = -17,265$ ✓

(b) Energy conservation on C.V. for T_3

$$\dot{E}_{in} = -17,265\ [W]$$

$$\dot{E}_{gen} = 1600\ [W]$$

$$\dot{E}_{out} = h_\infty A (T_R - T_\infty)$$

$$\dot{E}_{in} + \dot{E}_{gen} = -17,265 + 1600 = -15,665\ [W]$$

$$\Rightarrow \quad 40\left[\frac{W}{m^2 K}\right] 1\ [m^2] (T_R - 800)\ [K] = -15,665\ [W]$$

$$40\,T_R - 40(800) = -15,665$$

$$T_R = \frac{-15,665 + (40)(800)}{40} = \underline{408.38\ [K]}$$

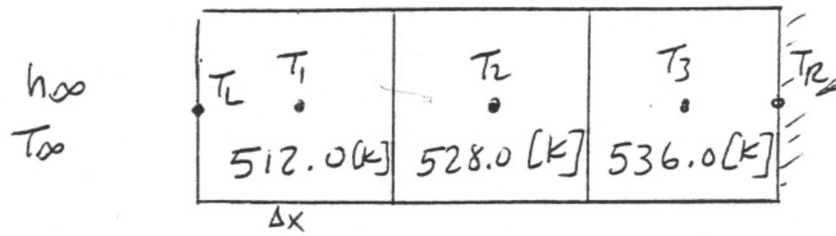
(c) Overall Energy Conservation

$\dot{E}_{in} = -20,460\ [W]$ $\quad$ from part (a)

$$\dot{E}_{gen} = 3 \cdot 1600 = 4800\ [W]$$

$$\dot{E}_{in} + \dot{E}_{gen} = -20,460 + 4800 = -15,660\ [W]$$

$$\Rightarrow \quad T_R = \frac{-15,660 + 40(800)}{40} = \underline{408.50\ [K]}$$



$\Delta x = \frac{L}{3} = \frac{0.6}{3} = 0.2\ [m]$

$h_\infty = 60\ [W/m^2 K]$
$T_\infty = 300\ [K]$

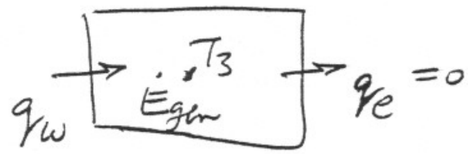
$k = 100\ [W/m\,K]$
$\dot{Q}''' = 20{,}000\ [W/m^3]$
$\Delta x = 0.2\ [m]$

(a) By inspection, because of right boundary condition

$\frac{dT}{dx} = 0 \qquad T_3 = T_R \qquad \Rightarrow \quad \boxed{T_R = 536.0\ [K]}$

(b) Energy balance on c.v. for T_3

$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$



$$q_w + \dot{Q}'''(Vol) \stackrel{?}{=} q_e$$

$$-kA\frac{dT}{dx}\bigg|_w + \dot{Q}''' A(\Delta x) \stackrel{?}{=} 0$$

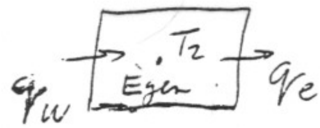
$$-100(1)\frac{(536-528)}{0.2} + 20{,}000(1)\,0.2 \stackrel{?}{=} 0$$

$$-4000 \quad + \quad 4000 \quad = 0 \quad \checkmark$$

Energy is conserved.

(c) Energy balance for C.V. for T_2

$$q_w + \dot{Q}'''(Vol) = q_e$$



but q_e for T_2 C.V. is q_w for T_3 CV which we already calculated. Also, the $\dot{E}_{gen}$ term is the same as before.

$$-kA\left(\frac{T_2 - T_1}{\Delta x}\right) + \dot{Q}'''A(\Delta x) = -k\left(\frac{T_3 - T_2}{\Delta x}\right)$$

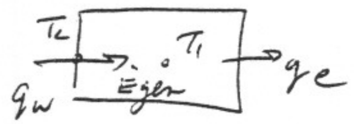
$$-100(1)\left(\frac{528.0 - 512.0}{0.2}\right) + 4000 \stackrel{?}{=} -4000$$

$$-8000 + 4000 = -4000$$

$$-4000 = -4000 \checkmark$$

Energy is conserved

(d) Energy balance on CV: for T_1



$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$$

$$h_\infty A(T_\infty - T_L) + \dot{Q}'''(Vol) = -kA\left(\frac{T_2 - T_1}{\Delta x}\right)$$

$$h_\infty A(T_\infty - T_L) + 4000 = -8000$$

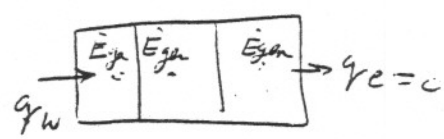
$$h_\infty (1)(T_\infty - T_L) = -12{,}000$$

$$\Rightarrow \quad T_L = T_\infty + \frac{12{,}000}{h_\infty}$$

$$T_L = 300 + \frac{12{,}000}{60} = 500.0$$

$$\boxed{T_L = 500.0\ [K]}$$

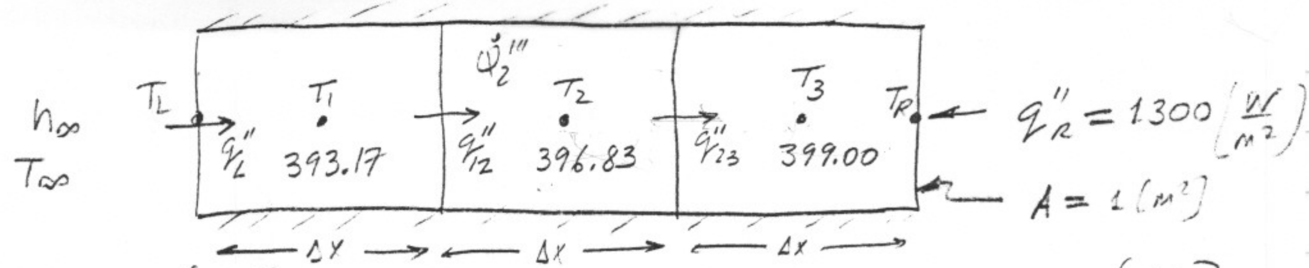
(e) From overall energy conservation



$$h_\infty A(T_\infty - T_L) + 3(\dot{Q}''')Vol = 0$$

$$60(1)(300 - T_L) + 3(20{,}000)(1)(0.2) = 0$$

$$\Rightarrow \boxed{T_L = 500.0\ [K]}$$



$h_\infty = 120 \left[\frac{W}{m^2 K}\right]$ $\qquad \Delta x = \frac{0.3}{3}\,[m] = 0.1\,[m]$ $\qquad k = 60 \left[\frac{W}{mK}\right]$

$T_\infty = 373\,[K]$

$\dot{Q}_2''' = 9000 \left(\frac{W}{m^3}\right)$

(a) T_3 control volume

$$\dot{E}_{in} + \dot{E}_{gen} \stackrel{?}{=} \dot{E}_{out}$$

$$q_{23}'' \cdot A + q_R'' \cdot A + 0 \stackrel{?}{=} 0$$

$$-k\frac{(T_3 - T_2)}{\Delta x} \cdot A + q_R'' \cdot A \stackrel{?}{=} 0$$

$$\frac{-60\,(399.00 - 396.83) \cdot 1}{0.1} + 1300 \cdot 1 \stackrel{?}{=} 0$$

$$-1302 + 1300 \stackrel{?}{=} 0$$

$$-2 \stackrel{?}{=} 0$$

→ Given the number of significant figures for the T values, this does balance. (Yes, it balances to within $\frac{2}{1300} \simeq 0.15\,\%$)

(b) T_2 Control volume

$$\dot{E}_{in} + \dot{E}_{gen} \stackrel{?}{=} \dot{E}_{out}$$

$$q_{12}'' A + \dot{Q}_2''' A \cdot \Delta x \stackrel{?}{=} q_{23}'' \cdot A$$

$$-k\frac{(T_2 - T_1)}{\Delta x} A + \dot{Q}_2''' A\,\Delta x \stackrel{?}{=} -k\frac{(T_3 - T_2)}{\Delta x} A$$

CONTINUED

5.3 (CONTINUED)

$$-\frac{60(396.83-393.17)}{0.1}\,1 + 9000\cdot 1\cdot 0.1 \overset{?}{=} -1302$$

$$-2196 \quad + \quad 900 \quad \overset{?}{=} -1302$$

$$\boxed{-1296 \quad \overset{?}{=} \quad -1302}$$

This balances within $\frac{6}{1302} = 0.46\%$.

(c) Energy balance on T_1 control volume:

$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$$

$$h_\infty(T_\infty - T_L)A + 0 = -k\frac{(T_2-T_1)}{\Delta x}A$$

$$\Rightarrow \quad T_L = \frac{1}{h_\infty}\left\{ k\frac{(T_2-T_1)}{\Delta x} + h_\infty T_\infty \right\}$$

$$T_L = \frac{1}{120}\left\{ \frac{60(396.83-393.17)}{0.1} + 120(373) \right\}$$

$$T_L = 391.30\ [K]$$

CONTINUED

(d) Overall energy balance

$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$$

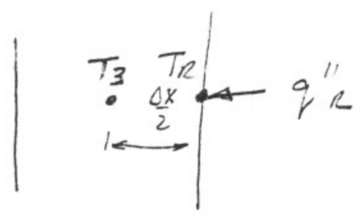
$$h_\infty (T_\infty - T_L) A + q''_R \cdot A + \dot{Q}_2''' A \cdot \Delta x = 0$$

$$T_L = \frac{1}{h_\infty} \left\{ h_\infty T_\infty + q''_R + \dot{Q}_2''' \Delta x \right\}$$

$$T_L = \frac{1}{120} \left\{ 120\,(373) + 1300 + 9000(0.1) \right\}$$

$$T_L = 391.33\ [K]$$

e) T_R can be calculated from an energy balance on the right boundary



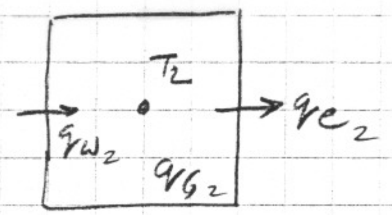
$$-k \left.\frac{dT}{dx}\right|_R = -q''_R$$

$$\frac{-60\,(T_R - T_3)}{\left(\frac{\Delta x}{2}\right)} = -1300$$

$$T_R = 1300\left(\frac{0.1}{2}\right)\frac{1}{60} + 399.00 = \underline{\underline{400.08\ [K]}}$$

5.4

(a) C.V. for T_2



Energy balance $q_{w_2} + q_{G_2} = q_{e_2}$ ①

$$q_{w_2} = -kA\frac{(T_2 - T_1)}{\Delta x} = -15(1)\frac{(427.12 - 420.17)}{0.15} = -695.00 [W]$$

$$q_{G_2} = \dot{Q}_2''' A \Delta x = (6000)(1)(0.15) = 900 [W]$$

$$q_{e_2} = -kA\frac{(T_3 - T_2)}{\Delta x} = -15(1)\frac{(425.06 - 427.12)}{0.15} = +206.00$$

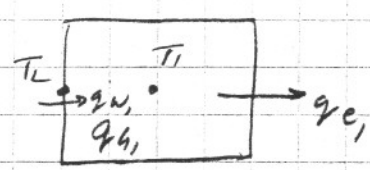
Eqn ①: $-695 + 900 \stackrel{?}{=} 206$

$205 \doteq 206$ ✓ balances within 0.5%

(b) C.V. for T_1

$h_{\infty L} = 10 \left[\frac{W}{m^2 K}\right]$

$T_{\infty L} = 300 [K]$



$= -695 [W]$

Energy balance $q_{w_1} + q_{G_1} = q_{e_1}$

$$h_{\infty L} A (T_{\infty L} - T_L) + \dot{Q}_1''' A (\Delta x) = q_{e1}$$

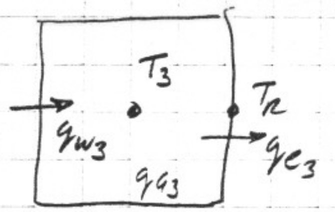
$$(T_{\infty L} - T_L) + \frac{\dot{Q}_1''' A \Delta x}{h_{\infty L} A} = \frac{q_{e1}}{h_{\infty L} A}$$

$$T_L = T_{\infty L} + \frac{\dot{Q}_1''' A \Delta x}{h_{\infty} A} - \frac{q_{e1}}{h_{\infty L} A} = 300 + \frac{3000 (1)\, 0.15}{10(1)} - \frac{(-695)}{10(1)}$$

$$T_L = 300 + 45 + 69.5 = \underline{\underline{414.5 [K]}}$$

CONTINUED

(c) C.V. for T_3



$h_{\infty R} = 90 \; [W/m^2K]$

$T_{\infty R} = 400 \; [K]$

Energy balance: $q_{w_3} + q_{G_3} = q_{e_3}$

$$q_{w_3} + \dot{Q}_3''' A(\Delta x) = h_{\infty R} A (T_R - T_{\infty R})$$

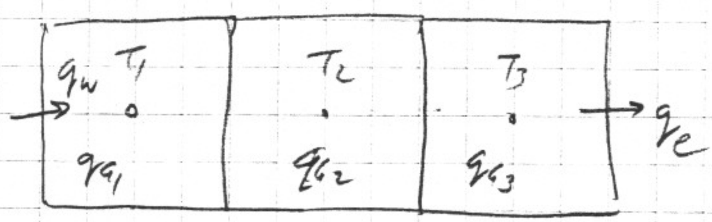
$$\frac{q_{w_3}}{h_{\infty R} A} + \frac{\dot{Q}_3''' A(\Delta x)}{h_{\infty R} A} = (T_R - T_{\infty R})$$

$$T_R = T_{\infty R} + \frac{q_{w_3}}{h_{\infty R} A} + \frac{\dot{Q}''' A(\Delta x)}{h_{\infty R} A}$$

$$T_R = 400 + \frac{206}{90(1)} + \frac{9000(1)(0.15)}{90(1)}$$

$$T_R = 400 + 2.2889 + 15.00 = \underline{\underline{417.29}} \; [K]$$

(d) overall



$$q_w + q_{G_1} + q_{G_2} + q_{G_3} = q_e$$

$$h_{\infty L} A (T_{\infty L} - T_L) + \dot{Q}_1''' A \Delta x + \dot{Q}_2''' A \Delta x + \dot{Q}_3''' A \Delta x = h_\infty A (T_R - T_{\infty R})$$

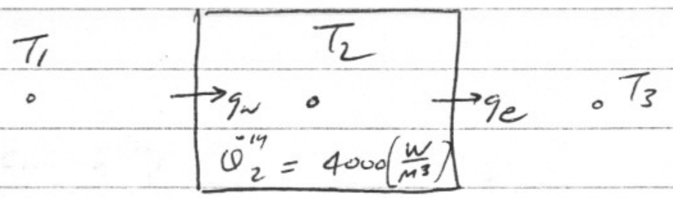
$$10(1)(300 - 414.5) + 3000(1)(0.15) + 6000(1)\,0.15 + 9000(1)(0.15) \stackrel{?}{=} 90(1)(417.29 - 400)$$

$$-1145 + 450 + 900 + 1350 \stackrel{?}{=} 1556.1$$

$$1555 \stackrel{?}{=} 1556.1 \; \checkmark$$ (balances within 0.07 %)

5.5

(a)



$\Delta x = 0.20$ [m]

$A = 1$ [m²]

$\frac{dT}{dx} \approx \frac{\Delta T}{\Delta x}$

$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$$

$$q_w + \dot{Q}'''_2 (Vol)_2 = q_e$$

$$q_w = -k_w A_w \left.\frac{dT}{dx}\right|_w = -\frac{(12)(1)(483.33 - 463.33)}{0.20} = -1200. \text{ [W]}$$

$$\dot{Q}'''_2 (Vol)_2 = (4000)(0.20)(1) = 800 \text{ [W]}$$

$$q_e = -k_e A_e \left.\frac{dT}{dx}\right|_e = -(12)(1)\frac{(490.00 - 483.33)}{0.20} = -400.20 \text{ [W]}$$

$$-1200 + 800 \overset{?}{=} -400.20$$

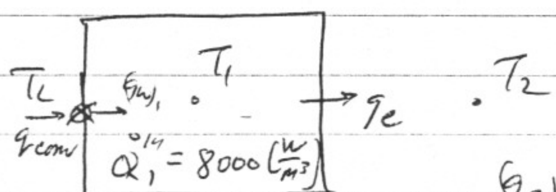
$$-400 = -400.20 \checkmark$$

balances within

$$\frac{0.20}{400} = 0.05\ \%$$

(b) $h_{\infty L} = 20 \left(\frac{W}{m^2 K}\right)$

$T_{\infty L} = 300$ [K]



$(q_e)_1 = (q_w)_2 = -1200$ (W)

Energy balance at the left boundary

$$q_{conv} = (q_w)_1$$

$$h_{\infty L} \cancel{(A)} (T_{\infty L} - T_L) = -k_w A_w \frac{(T_1 - T_L)}{\frac{\Delta x}{2}}$$

CONTINUED

$$\frac{h_{\infty_L}}{k_w}\left(\frac{\Delta x}{2}\right)\left(T_{\infty_L} - T_L\right) = -\left(T_1 - T_L\right)$$

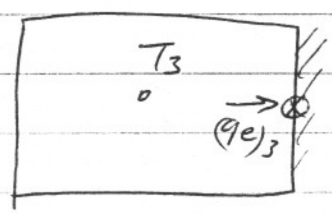
$$\left(1 + \frac{h_{\infty_L}}{k_w}\left(\frac{\Delta x}{2}\right)\right) T_L = \left(\frac{h_{\infty_L}}{k_w}\frac{\Delta x}{2}\right) T_{\infty_L} + T_1$$

$$\left(1 + \frac{(20)}{12}\frac{(0.20)}{2}\right) T_L = \frac{20\,(0.20)}{12\,(2)}\left(300\right) + 463.33$$

$$(1.16667)\, T_L = 513.33$$

$$T_L = 440.00 \; [K] \leftarrow$$

(c)



$$q_{e_3} = -k_e A_e \left.\frac{dT}{dx}\right|_e = -k_e A_e \frac{(T_R - T_3)}{\frac{\Delta x}{2}} = 0 \qquad \text{insulated}$$

$$\Rightarrow (T_R - T_3) = 0$$

$$T_R = T_3 = 490.00 \; [K] \leftarrow$$

Comparison of the 2D steady conduction equation with the general equation indicates $\Gamma = k$, $u = v = 0$. There is no source term in this problem $\rightarrow \dot{S}''' = 0$.

The coefficients of the algebraic equation are therefore ($\dot{m} = 0$), using uniform grid spacing:

$$a_E = D_e = \frac{\Gamma_e A_e}{(\delta x)_e} = \frac{k_e A_e}{(\delta x)_e} = \frac{k_e \, \Delta Y \cdot 1}{\Delta x}$$

$$a_W = D_w = \frac{\Gamma_w A_w}{(\delta x)_w} = \frac{k_w A_w}{(\delta x)_w} = \frac{k_w \, \Delta Y \cdot 1}{\Delta x}$$

$$a_N = D_n = \frac{\Gamma_n A_n}{(\delta y)_n} = \frac{k_n A_n}{(\delta y)_n} = \frac{k_n \, \Delta X \cdot 1}{\Delta Y}$$

$$a_S = D_s = \frac{\Gamma_s A_s}{(\delta y)_s} = \frac{k_s A_s}{(\delta y)_s} = \frac{k_s \, \Delta X \cdot 1}{\Delta Y}$$

[$\beta = 1$ because it is conduction (no flow)].

Using the harmonic mean and the nodal properties from the diagram:

$$k_e = \frac{2 k_P k_E}{(k_P + k_E)} = \frac{2(0.20)(0.20)}{(0.20 + 0.20)} = 0.20 \left[\frac{W}{mK}\right]$$

CONTINUED

$$k_w = \frac{2 k_W k_P}{(k_W + k_P)} = \frac{2(15.0)(0.20)}{(15.0 + 0.20)} = 0.3947 \left[\frac{W}{mK}\right]$$

$$k_n = 2\frac{k_P k_N}{(k_P + k_N)} = \frac{2(0.20)(0.20)}{(0.20 + 0.20)} = 0.20 \left[\frac{W}{mK}\right]$$

$$k_s = \frac{2 k_s k_P}{(k_s + k_P)} = \frac{2(15.0)(0.20)}{(15.0 + 0.20)} = 0.3947 \left[\frac{W}{mK}\right]$$

$$a_E = \frac{(0.20)(0.10)\,1}{0.05} = 0.40$$

$$a_W = \frac{(0.3947)(0.10)\,1}{0.05} = 0.7894$$

$$a_N = \frac{(0.20)(0.05)\,1}{0.10} = 0.10$$

$$a_S = \frac{(0.3947)(0.05)\,1}{0.10} = 0.19735$$

$$a_P = \Sigma a_{NP} = 0.40 + 0.7894 + 0.10 + 0.19735$$

$$a_P = 1.48675$$

$$b_P = \dot{S}'''(Vol)_P = 0 \cdot (Vol)_P = 0.0$$

(b) The algebraic equation for T_P is therefore

$$1.48675\, T_P = 0.40\, T_E + 0.7894\, T_W + 0.10\, T_N + 0.19735\, T_S$$

For the neighbour temperatures given,

$$T_P = \frac{1}{1.48675}\left\{0.40(400) + 0.7894(300) + 0.10(420) + 0.19735(320)\right\}$$

$$T_P = 337.6\,[K]$$

(a) Coefficients for the algebraic equation for T_6:

$$a_E = D_e = \frac{\Gamma_e A_e}{(\delta x)_e} = \frac{k_e \Delta y\, 1}{\Delta x}$$

$$k_e = \frac{2 k_6 k_7}{(k_6 + k_7)} = \frac{2 k_A k_B}{(k_A + k_B)} = \frac{2\,(24)\,(2)}{(24+2)} = 3.6923 \left(\frac{W}{mK}\right)$$

$$\Rightarrow a_E = 3.6923 \cdot \frac{0.02}{0.05}\, 1 = \underline{\underline{1.47692}} \quad \leftarrow a_{E_6}$$

$$a_S = D_s = \frac{\Gamma_s A_s}{(\delta y)_s} = \frac{k_s \Delta x \cdot 1}{\Delta y}$$

$$k_s = \frac{2 k_6 k_2}{(k_6 + k_2)} = \frac{2 k_A k_C}{(k_A + k_C)} = \frac{2\,(24)\,0.1}{(24 + 0.1)} = 0.19917$$

$$a_S = 0.19917\ \frac{0.05}{0.02}\, 1 = \underline{0.49793} \quad \leftarrow a_{S_6}$$

$$a_W = D_w = \frac{k_w A_w}{(\delta x)_w} = \frac{k_w \Delta y \cdot 1}{\Delta x} = k_A \cdot \frac{\Delta y}{\Delta x} \cdot 1 = \frac{24\,(0.02)\,1}{0.05}$$

$$a_W = \underline{9.6} \quad \leftarrow a_{W_6}$$

$$a_N = D_n = \frac{k_n A_n}{(\delta y)_n} = \frac{k_n \Delta x\, 1}{\Delta y} = k_A \frac{\Delta x\, 1}{\Delta y} = \frac{24\,(0.05)\,1}{0.02}$$

$$a_N = \underline{60} \quad \leftarrow a_{N_6}$$

$$a_P = \Sigma a_{NB} = a_E + a_W + a_N + a_S = 1.47692 + 9.6 + 60 + 0.49793$$

$$a_P = 71.575 \quad \leftarrow a_{P6}$$

$$b_P = 0.0 \quad \text{(No energy generation)} \quad \leftarrow b_{P6}$$

CONTINUED

(b) Coefficients for the algebraic equation for T_7

$$k_e = k_B = 2.0 \left[\frac{W}{mK}\right]$$

$$k_n = k_B = 2.0 \left(\frac{W}{mK}\right)$$

$$k_w = \frac{2 k_7 k_6}{(k_7 + k_6)} = \frac{2(2)(24)}{(2+24)} = 3.6923 \left[W/mK\right]$$

$$k_s = \frac{2 k_7 k_3}{(k_7 + k_3)} = \frac{2(2)(0.1)}{(2+0.1)} = 0.19048 \left(W/mK\right)$$

$$a_E = \frac{k_e A_e}{(\delta x)_e} = \frac{k_e \Delta y \cdot 1}{\Delta x} = \frac{2.0(0.02)\,1}{0.05} = 0.8 \quad \leftarrow a_{E_7}$$

$$a_W = \frac{k_w A_w}{(\delta x)_w} = \frac{k_w \Delta y \cdot 1}{\Delta x} = \frac{3.6923 \cdot (0.02)\,1}{0.05} = 1.47692 \leftarrow a_{W_7}$$

$$a_N = \frac{k_n A_n}{(\delta y)_n} = \frac{k_n \Delta x \cdot 1}{\Delta y} = \frac{2.0(0.05)\,1}{0.02} = 5.0 \quad \leftarrow a_{N_7}$$

$$a_S = \frac{k_s A_s}{(\delta y)_s} = \frac{k_s \Delta x \cdot 1}{\Delta y} = \frac{0.19048\,(0.05)\,1}{0.02} = 0.47620 \leftarrow a_{S_7}$$

$$a_P = \Sigma a_{NP} = a_E + a_W + a_N + a_S = 0.8 + 1.47692 + 5.0 + 0.47620$$

$$a_P = 7.7531 \quad \leftarrow a_{P_7}$$

$b_P = 0$ (no energy generation) $\leftarrow b_{P_7}$

(c) Equation from part (a)

$$a_{P_6} T_6 = a_{E_6} T_7 + a_{W_6} T_5 + a_{N_6} T_{10} + a_{S_6} T_2$$

CONTINUED

$$71.575\, T_6 = 1.47692\, T_7 + 9.6\, T_5 + 60\, T_{10} + 0.49793\, T_2$$

$T_2 = 400 \qquad T_5 = 320 \qquad T_{10} = 320$

$$T_6 = \left(\frac{1.47692}{71.575}\right) T_7 + \frac{\{9.6(320) + 60(320) + 0.49793(400)\}}{71.575}$$

$$\boxed{T_6 = 0.020635\, T_7 + 313.953} \qquad (1)$$

Equation from part (b)

$$a_{P_7} T_7 = a_{E_7} T_8 + a_{W_7} T_6 + a_{N_7} T_{11} + a_{S_7} T_3$$

$$7.7531\, T_7 = 0.8\, T_8 + 1.47692\, T_6 + 5.0\, T_{11} + 0.47620\, T_3$$

$T_8 = 500 \qquad T_{11} = 360 \qquad T_3 = 500$

$$T_7 = \left(\frac{1.47692}{7.7531}\right) T_6 + \frac{\{0.8(500) + 5.0(360) + 0.47620(500)\}}{7.7531}$$

$$\boxed{T_7 = 0.19049\, T_6 + 314.467} \qquad (2)$$

CONTINUED

(d) Substitute Equation (2) into Equation (1)

$$T_6 = 0.020635\left\{0.19049\,T_6 + 314.467\right\} + 313.953$$

$$\Rightarrow \quad T_6 = \frac{\left[0.020635(314.467) + 313.953\right]}{\left[1 - 0.020635(0.19049)\right]}$$

$$T_6 = 321.71\ [K]$$

$$T_7 = 0.19049(321.71) + 314.467$$

$$T_7 = 375.75\ [K]$$

Summary

$$\boxed{\begin{aligned} T_6 &= 321.7\ [K] \\ T_7 &= 375.8\ [K] \end{aligned}}$$

(a) C.V. for T6

$$(a_W)_6 = \frac{k_W A_W}{(\delta x)_W}$$

$(k_W)_6 = k_A = 16 \, [W/mK]$
$(A_W)_6 = 0.3(1)\,[m] = 0.3\,[m^2]$
$(\delta x)_W = 0.6 \; [m]$

$$(a_W)_6 = \frac{16\,(0.3)}{0.6} = 8.000$$

$$(a_S)_6 = \frac{k_S A_S}{(\delta y)_S}$$

$(k_S)_6 = k_A = 16 \; [W/mK]$
$A_S = 0.4(1) = 0.4\,[m^2]$
$(\delta y)_S = 0.3\,[m]$

$$(a_S)_6 = \frac{16\,(0.4)}{0.3} = 21.333$$

$$(a_E)_6 = \frac{k_e A_e}{(\delta x)_e} \qquad (k_e)_6 = \frac{k_E k_P}{f_e k_P + (1-f_e)\,k_E}$$

$k_P = k_A$
$k_E = k_C$
$f_e = 0.33333$

$$(k_e)_6 = \frac{k_A k_C}{[f_e k_A + (1-f_e)\,k_C]}$$

$$(k_e)_6 = \frac{(16)(64)}{[0.33333(16) + (1-0.33333)\,64]} = 21.333 \left(\frac{W}{mK}\right)$$

$(A_e)_6 = 0.3(1) = 0.3 \; (m^2)$
$\delta y_e = 0.30\,[m]$

$$(a_E)_6 = (21.333)\frac{(0.30)}{(0.30)} = 21.333$$

$$(a_N)_6 = \frac{k_n A_n}{(\delta y)_n} \qquad (k_n)_6 = \frac{k_P k_N}{[f_n k_P + (1-f_n)\,k_N]}$$

$k_P = k_A$ $\qquad f_n = 0.62500$
$k_N = k_B$

CONTINUED

5.8 (CONTINUED)

$$(k_n)_6 = \frac{k_A k_B}{f_n k_A + (1-f_n) k_B} = \frac{(16)(4)}{[0.625(16) + (1-0.625)4]}$$

$$(k_n)_6 = 5.5652 \left(\frac{W}{mK}\right)$$

$(A_n)_6 = 0.4(1) = 0.4 \ (m^2)$

$(\delta y)_n = 0.40 \ [m]$

$$(a_N)_6 = \frac{(5.5652)(0.4)}{(0.4)} = 5.5652$$

$$(a_P)_6 = \Sigma (a_{NP})_6 = 8.0000 + 21.333 + 21.333 + 5.5652 = \underline{\underline{56.2312}}$$

$(b_P)_6 = 0.0$ (NO source: $q''' = 0$

(b) C.V. for T_7

$$(a_W)_7 = (a_E)_6 = 21.333$$

$$(a_E)_7 = \frac{k_e A_e}{\delta x)_e}$$

$k_e = k_c = 64 \ (W/mK)$

$A_e = 0.3(1) = 0.3 \ [m^2)$

$(\delta x)_e = 0.20 \ [m]$

$$(a_E)_7 = \frac{64(0.3)}{0.20} = 96.00$$

$$(a_S)_7 = \frac{k_s A_s}{(\delta y)_s}$$

$k_s = k_c = 64 \ (W/mK)$

$A_s = 0.2(1) = 0.2 \ (m^2)$

$(\delta y)_s = 0.30 \ [m]$

$$(a_S)_7 = \frac{(64)(0.20)}{(0.30)} = 42.667$$

CONTINUED