(CONTINUED)

$$(a_N)_7 = \frac{k_n A_n}{(\delta y)_n} \qquad k_n = \frac{k_P k_N}{f_n k_P + (1-f_n) k_N} \qquad k_P = k_C \quad k_N = k_B$$

$$k_n = \frac{k_C k_B}{[f_n k_C + (1-f_n) k_B]} \qquad f_n = 0.62500$$

$$(k_n)_7 = \frac{(64)(4)}{[(0.62500)64 + (1-0.62500)(4)]} = 6.1687 \left(\frac{W}{mK}\right)$$

$$A_n = 0.2(1) = 0.2 \,(m^2)$$

$$(a_n)_7 = \frac{6.1687\ (0.20)}{0.40} = \underline{3.0844} \qquad (\delta y)_n = 0.4 \,(m)$$

$$(a_P)_7 = 21.333 + 96.00 + 42.667 + 3.0844 = 163.0844$$

$$(b_P)_7 = 0.0$$

(c) $(a_P)_6 T_6 = (a_W)_6 T_5 + (a_E)_6 T_7 + (a_S)_6 T_2 + (a_N)_6 T_{10} + 0$

$$56.2312\, T_6 = 8.000(320) + 21.333 T_7 + 21.333(310) + 5.5652(380)$$

$$56.2312\, T_6 = 21.333\, T_7 + 11{,}288.0 \qquad (1)$$

$$(a_P)_7 T_7 = (a_W)_7 T_6 + (a_E)_7 T_8 + (a_S)_7 T_3 + (a_N)_7 T_{11} + 0$$

$$163.0844\, T_7 = 21.333\, T_6 + 96.00(370) + 42.667(340) + 3.0844(410)$$

$$163.0844\, T_7 = 21.333\, T_6 + 51{,}291.4 \qquad (2)$$

CONTINUED

(d) $$56.2312\, T_6 = 21.333\, T_7 + 11{,}288.0 \quad (1)$$

$$163.0844\, T_7 = 21.333\, T_6 + 51{,}291.4 \quad (2)$$

From (1)

$$T_6 = \frac{21.333\, T_7}{56.2312} + \frac{11{,}288.0}{56.2312}$$

$$T_6 = 0.37938\, T_7 + 200.743 \quad (3)$$

Substitute (3) into (2)

$$163.0844\, T_7 = 21.333\,(0.37938\, T_7 + 200.743) + 51{,}291.4$$

$$\left(163.0844 - 21.333(0.37938)\right) T_7 = 21.333(200.743) + 51{,}291.4$$

$$154.991\, T_7 = 55{,}573.85$$

$$T_7 = \underline{358.56} \; [K]$$

$$T_6 = 0.37938\,(358.56) + 200.743) = 336.77\,(K)$$

$$T_6 = 336.77\,(K)$$

$$T_7 = 358.56\,[K]$$

- 2D Steady conduction
  - no source term
  - composite
  - uniform grid spacing but $\Delta x \neq \Delta y$
  - ($A_n = A_s = \Delta x \quad A_e = A_w = \Delta y$)
  - $\Gamma = k$

Eqns (15.7) to (15.10) simplify in this case to

$$\left[\frac{k_e A_e}{(\delta x)_e} + \frac{k_w A_w}{(\delta x)_w} + \frac{k_n A_n}{(\delta y)_n} + \frac{k_s A_s}{(\delta y)_s}\right] T_P = \frac{k_e A_e}{(\delta x)_e} T_E + \frac{k_w A_w}{(\delta x)_w} T_W + \frac{k_n A_n}{(\delta y)_n} T_N + \frac{k_s A_s}{(\delta y)_s} T_S$$

$$a_P = a_E + a_W + a_N + a_S \qquad b_P = 0$$

(a) • CV for T_1 $\quad k_e = k_w = k_n = k_s = k_A = 2\ (W/mK)$

$$(\delta x)_w = \frac{\Delta x}{2} \qquad (\delta y)_s = \frac{\Delta y}{2}$$

$$(a_E)_1 = \frac{k_A \Delta y}{\Delta x} = 2\left(\frac{0.1}{0.4}\right) = 0.5 \qquad (a_N)_1 = \frac{k_A \Delta x}{\Delta y} = \frac{2(0.4)}{0.1} = 8$$

$$(a_W)_1 = \frac{k_A \Delta y}{\frac{\Delta x}{2}} = \frac{2(0.1)}{0.2} = 1.0 \qquad (a_S)_1 = \frac{k_A (\Delta x)}{\frac{\Delta y}{2}} = \frac{2(0.4)}{0.05} = 16$$

$$(a_P)_1 = 0.5 + 1.0 + 8 + 16 = 25.5$$

B.C : $T_W = T_L \qquad T_S = T_1$ (by zero gradient)

$$25.5\, T_1 = 0.5\, T_2 + T_L + 8\, T_5 + 16\, T_1$$

$$\boxed{9.5\, T_1 = 0.5\, T_2 + T_L + 8\, T_5}$$

• CV for T_2 $\quad a_{W_2} = a_{E_1} = 0.5$

by inspection $\quad a_{S_2} = a_{S_1} = 16$

$$(k_n)_2 = \frac{2\, k_A k_B}{(k_A + k_B)} = \frac{2(2)(48)}{(2+48)} = 3.84\ (W/mK)$$

$$(k_e)_2 = \frac{2(k_A k_B)}{(k_A + k_B)} = 3.84\ (W/mK)$$

CONTINUED

$$(a_N)_2 = \frac{(3.84)(0.4)}{0.1} = 15.36$$

$$(a_E)_2 = \frac{(3.84)(0.1)}{(0.4)} = 0.96$$

$$a_P)_2 = 0.5 + 0.96 + 15.36 + 16 = 32.82$$

insulated both $T_5 = T_2$

$$32.82\, T_2 = 0.5\, T_1 + 0.96\, T_3 + 15.36\, T_6 + 16\, T_2$$

$$\boxed{16.82\, T_2 = 0.5\, T_1 + 0.96\, T_3 + 15.36\, T_6}$$

(b) Substitute known T values

$$9.5\, T_1 = 0.5\, T_2 + 500 + 8(425.70)$$

$$\boxed{9.5\, T_1 = 0.5\, T_2 + 3905.60} \qquad (3\text{-}1)$$

$$16.82\, T_2 = 0.5\, T_1 + 0.96(359.00) + 15.36\,(382.06)$$

$$\boxed{16.82\, T_2 = 0.5\, T_1 + 6213.1} \qquad (3\text{-}2)$$

(c) Solve Eqns (3-1) & (3-2)

From (3-1) $\quad T_1 = \frac{0.5}{9.5} T_2 + \frac{3905.60}{9.5} \qquad (3\text{-}3)$

Substitute (3-3) into (3-2)

$$16.82\, T_2 = 0.5\left(\frac{0.5}{9.5} T_2 + \frac{3905.60}{9.5}\right) + 6213.1$$

$$\left(16.82 - \frac{0.25}{9.5}\right) T_2 = \frac{0.5(3905.60)}{9.5} + 6213.1$$

$$16.793\, T_2 = 6418.7 \quad \rightarrow \underline{T_2 = 382.22} \ (K)$$

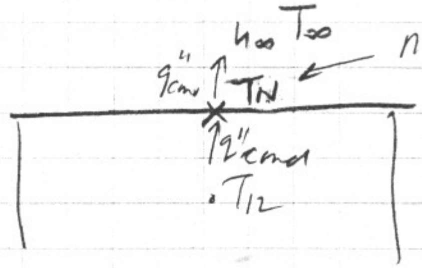
$$T_1 = \frac{0.5}{9.5}(382.22) + \frac{3905.60}{9.5} = \underline{431.23} \ (K)$$

CONTINUED

(d) cv for T_{12} $\quad (k_e)_{12} = (k_w)_{12} = (k_n)_{12} = k_C = 12 \ (W/mK)$

$$(k_s)_{12} = \frac{2 k_B k_C}{(k_B + k_C)} = \frac{2(48)(12)}{(48+12)} = 19.2 \ (W/mK)$$

Top boundary condition for 12



$$q''_{cond} = q''_{conv}$$

$$\frac{-k_C(T_N - T_{12})}{\Delta y/2} = h_\infty(T_N - T_\infty)$$

$$-k_C T_N + k_C T_{12} = \frac{h_\infty \Delta y}{2} T_N - \frac{h_\infty \Delta y}{2} T_\infty$$

$$\left(k_C + \frac{h_\infty \Delta y}{2}\right) T_N = k_C T_{12} + \frac{h_\infty \Delta y}{2} T_\infty$$

$$\boxed{T_N = \frac{k_C}{(k_C + \frac{1}{2} h_\infty \Delta y)} T_{12} + \frac{\frac{1}{2} h_\infty \Delta y}{(k_C + \frac{1}{2} h_\infty \Delta y)} T_\infty}$$

Use this equation for T_N in T_{12} equation

$$(a_E)_{12} = 12\left(\frac{0.1}{0.4}\right) = 3 \qquad (a_W)_{12} = 12\left(\frac{0.1}{0.2}\right) = 6$$

$$(a_S)_{12} = 19.2\left(\frac{0.4}{0.1}\right) = 76.8 \qquad (a_N)_{12} = 12\left(\frac{0.4}{0.05}\right) = 96$$

CONTINUED

$$a_P = 3 + 6 + 76.8 + 96 = 181.8$$

$$181.8\,T_{12} = 3\,T_{11} + 6T_R + 76.8\,T_8 + 96T_N$$

$$T_N = \frac{12}{\left(12 + \frac{1}{2}(40)(0.1)\right)} T_{12} + \frac{\frac{1}{2}(40)(0.1)}{\left[12 + \frac{1}{2}(40)(0.1)\right]} 400$$

$$T_N = 0.85714\,T_{12} + 57.1429$$

$$181.8\,T_{12} = 3(366.69) + 6(300) + 76.8(330.28) + 96(0.85714\,T_{12} + 57.1429)$$

$$\Rightarrow \quad 99.515\,T_{12} = 33751.29$$

$$T_{12} = \underline{339.16\ [K]}$$

5.10

(a) $$T_L = T_1 + \frac{q_L'' \Delta x}{2k}$$

$$T_L = T_1 + \frac{(1000)(0.2)}{2(35)} = T_1 + 2.85714$$

$$\boxed{T_L = T_1 + 2.85714}$$

(b) Substitute T_L equation into previous algebraic equation for T_1

$$525\,T_1 = 350\{T_1 + 2.85714\} + 175\,T_2 + 111$$

$$525\,T_1 = 350\,T_1 + 1000 + 175\,T_2 + 111$$

$$\boxed{175\,T_1 = 175\,T_2 + 1111}$$

(c) $$T_R = \frac{1}{\left(1 + \frac{h_\infty \Delta x}{2k}\right)} T_3 + \frac{\left(\frac{h_\infty \Delta x}{2k}\right)}{\left(1 + \frac{h_\infty \Delta x}{2k}\right)} T_\infty$$

$$T_R = \frac{1}{\left(1 + \frac{70\,(0.2)}{2(35)}\right)} T_3 + \frac{\frac{70\,(0.2)}{2(35)}}{\left(1 + \frac{70\,(0.2)}{2(35)}\right)} 300$$

$$T_R = \frac{1}{1.2} T_3 + \frac{0.2}{1.2} 300$$

$$\boxed{T_R = 0.8333\,T_3 + 50}$$

CONTINUED

(d) substitute T_R equation into previous algebraic equation for T_3

$$525\,T_3 = 175\,T_2 + 350\left\{0.8333\,T_3 + 50\right\} + 111$$

$$525\,T_3 = 175\,T_2 + 291.666\,T_3 + 17{,}500 + 111$$

$$\boxed{233.33\,T_3 = 175\,T_2 + 17{,}611}$$

(e) The new equation set is

$$175\,T_1 = 175\,T_2 + 1111 \quad (1)$$

$$350\,T_2 = 175\,T_1 + 175\,T_3 + 111 \quad (2)$$

$$233.33\,T_3 = 175\,T_2 + 17{,}611 \quad (3)$$

which in matrix form is

$$\begin{bmatrix} 175 & -175 & 0 \\ -175 & 350 & -175 \\ 0 & -175 & 233.33 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 1111 \\ 111 \\ 17{,}611 \end{bmatrix} \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array}$$

CONTINUED

Solving: Add rows 1 & 2:

$$175\,T_2 - 175\,T_3 = 1222 \quad (4)$$

add (4) + row 5:

$$58.33\,T_3 = 18{,}833 \quad (5)$$

$$T_3 = \underline{322.87}$$

$$\Rightarrow \quad T_2 = \frac{1222 + 175(322.87)}{175}$$

$$T_2 = \underline{329.85}$$

$$T_1 = \frac{175(329.85) + 1111}{175}$$

$$T_1 = \underline{336.20}$$

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 336.20 \\ 329.85 \\ 322.87 \end{Bmatrix}$$

(a) $k_e = k_w = k = 70\,[W/m\,K]$

$A_e = A_w = A = 1\,[m^2]$

$\Delta x = L/3 = 0.6/3 = 0.2\,[m]$

$T_P = T_1$ $(\delta x)_w = \frac{\Delta x}{2} = 0.1\,[m]$

$T_W = T_L$

$T_E = T_2$ $(\delta x)_e = \Delta x = 0.2\,[m]$

Equation 2 $(i=1)$

$$\left(\frac{70(1)}{0.1} + \frac{70(1)}{0.2}\right) T_1 = \frac{70(1)}{0.1} T_L + \frac{70(1)}{0.2} T_2 + 1110(1)(0.2)$$

$$1050\,T_1 = 700\,T_L + 350\,T_2 + 222$$

$i=2$ $T_P = T_2$ $(T_W = T_1 \quad T_E = T_3)$

$$\left(\frac{70(1)}{0.2} + \frac{70(1)}{0.2}\right) T_2 = \frac{70(1)}{0.2} T_1 + \frac{70(1)}{0.2} T_3 + 1110(1)\,0.2$$

$$700\,T_2 = 350\,T_1 + 350\,T_3 + 222$$

$i=3$ $T_P = T_3$ $(T_W = T_2 \quad T_E = T_R)$

$$\left(\frac{70(1)}{0.2} + \frac{70(1)}{0.1}\right) T_3 = \frac{70(1)}{0.2} T_2 + \frac{70(1)}{0.1} T_R + 1110(1)(0.2)$$

$$1050\,T_3 = 350\,T_2 + 700\,T_R + 222$$

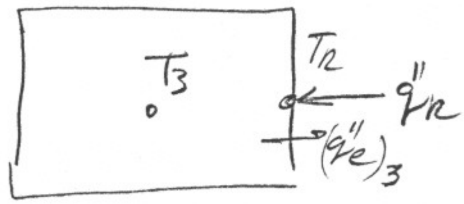
$$1050\,T_1 = 700\,T_L + 350\,T_2 + 222$$

$$700\,T_2 = 350\,T_1 + 350\,T_3 + 222$$

$$1050\,T_3 = 350\,T_2 + 700\,T_R + 222$$

CONTINUED

(b) Background
At the right boundary



$$(q''_e)_3 = -k_e \frac{dT}{dx}\bigg|_e = -k\frac{(T_R - T_3)}{\left(\frac{\Delta x}{2}\right)} = -\frac{2k}{\Delta x}(T_R - T_3)$$

But $q''_R = -(q''_e)_3$ so $q''_R = +\frac{2k}{\Delta x}(T_R - T_3)$

$$\Rightarrow \quad \frac{q''_R \Delta x}{2k} = T_R - T_3$$

$$\Rightarrow \quad \boxed{T_R = T_3 + \frac{q''_R \Delta x}{2k}}$$

Specific equation $T_R = T_3 + \frac{2000(0.2)}{2(70)}$

$$T_R = T_3 + 2.8571$$

(c) Substitute $T_L = 400$ in T_1 equation

$$1050\,T_1 = 700(400) + 350\,T_2 + 222$$

$$\Rightarrow \quad 1050\,T_1 = 350\,T_2 + 280222$$

Equation for T_2 is unchanged

Substitute for T_R in T_3 equation:

$$1050\,T_3 = 350\,T_2 + 700(T_3 + 2.8571) + 222$$

$$\Rightarrow \quad 350\,T_3 = 350\,T_2 + 2222$$

Modified equation set

$$1050\, T_1 = 350\, T_2 + 280222 \quad (1)$$

$$700\, T_2 = 350\, T_1 + 350\, T_3 + 222 \quad (2)$$

$$350\, T_3 = 350\, T_2 + 2222 \quad (3)$$

<u>Solution</u>

Eqn (1) − Eqn (3)

$$1050\, T_1 - 350\, T_3 = 278{,}000 \quad (4)$$

Eqn (2) + 2 Eqn (3)

$$\cancel{700 T_2} + 700\, T_3 = 350\, T_1 + 350 T_3 + 222 + \cancel{700 T_2} + 4444$$

$$-350\, T_1 + 350\, T_3 = 4666 \quad (5)$$

Eqn (4) + Eqn (5)

$$700\, T_1 = 282{,}666$$

$$T_1 = 403.81 \ [K]$$

$$\Rightarrow \quad T_3 = \frac{1}{350}\left(4666 + 350\,(403.81)\right)$$

$$T_3 = 417.14 \ [K]$$

$$T_2 = \frac{1}{700}\left(350\,(403.81) + 350\,(417.14) + 222\right)$$

$$T_2 = 410.79 \ [K]$$

CONTINUED

CHECK

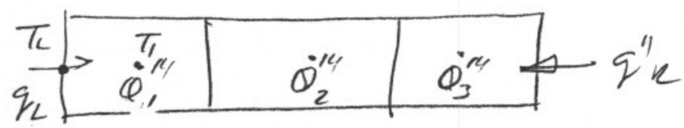
$$1050\,(403.81) \stackrel{?}{=} 350\,(410.79) + 280222 \quad \checkmark$$

$$700\,(410.79) \stackrel{?}{=} 350\,(403.81) + 350\,(417.14) + 222 \quad \checkmark$$

$$350\,(417.14) \stackrel{?}{=} 350\,(410.79) + 2222 \quad \checkmark$$

(d)

Overall energy balance:



$$\dot{E}_{in} + \dot{E}_{gen} = \dot{E}_{out}$$

$$q_L'' A + \left[\dot{Q}_1''' A\,\Delta x + \dot{Q}_2''' A\,\Delta x + \dot{Q}_3''' A\,\Delta x\right] = -q_R'' A$$

$$-k\frac{(T_1 - T_L)}{\left(\frac{\Delta x}{2}\right)} + \left(\dot{Q}_1''' + \dot{Q}_2''' + \dot{Q}_3'''\right)\Delta x = -q_R''$$

$$\frac{-70\,(403.81 - 400)}{0.1} + (3330)\,(0.2) \stackrel{?}{=} -2000$$

$$-2667 + 666 \stackrel{?}{=} -2000$$

$$-2001 \stackrel{?}{=} -2000$$

✓ balances to 0.05 %

5.12

(a) $$\underbrace{\int_t^{t+\Delta t}\int_{\forall} \frac{\partial}{\partial t}(\rho T)\, d\forall\, dt}_{\mathrm{I}} = \underbrace{\int_t^{t+\Delta t}\int_{\forall} \frac{\partial}{\partial x}\left(\Gamma\frac{\partial T}{\partial x}\right) d\forall\, dt}_{\mathrm{II}} + \underbrace{\int_t^{t+\Delta t}\int_{\forall} \frac{\partial}{\partial y}\left(\Gamma\frac{\partial T}{\partial y}\right) d\forall\, dt}_{\mathrm{III}}$$

$$\mathrm{I} = \int_t^{t+\Delta t}\int_{\forall}\left[\frac{\partial}{\partial t}(\rho T)\right]_p d\forall\, dt = \rho_p (\forall ol)_p \int_t^{t+\Delta t} \frac{\partial}{\partial t}(T_p)\, dt$$

(assuming $T \simeq T_p$ and ρ = constant

$$\boxed{\mathrm{I} = \rho_p (Vol)_p \left(T_p^{\,n} - T_p^{\,0}\right)}$$

$$\mathrm{II} = \int_t^{t+\Delta t}\int_A \Gamma\frac{\partial T}{\partial x}\Big|_e - \Gamma\frac{\partial T}{\partial x}\Big|_w \, dA\, dt = \int_t^{t+\Delta t}\left[\Gamma_e A_e \frac{\partial T}{\partial x}\Big|_e - \Gamma_w A_w \frac{\partial T}{\partial x}\Big|_w\right] dt$$

(assuming $\frac{\partial T}{\partial x}$ constant over each w & e face

$$\left.\begin{aligned} \frac{\partial T}{\partial x}\Big|_e &\simeq \frac{(T_E - T_p)}{(\delta x)_e} \\ \frac{\partial T}{\partial x}\Big|_w &\simeq \frac{(T_p - T_W)}{(\delta x)_w} \end{aligned}\right\}$$ piecewise linear profile assumed

$$\mathrm{II} = \int_t^{t+\Delta t}\left[\frac{\Gamma_e A_e}{(\delta x)_e}(T_E - T_p) - \frac{\Gamma_w A_w}{(\delta x)_w}(T_p - T_W)\right] dt$$

CONTINUED

5.12 (CONTINUED)

Using the Fully Explicit time weight directly Page 5-35/79

$(f_t = 0$ so $T_1 \simeq T^\circ) \Rightarrow \int_t^{t+\Delta t} T\,dt = T^\circ \Delta t$

$$\text{II} = \left[\frac{\Gamma_e A_e}{(\delta x)_e}\left(T_E^\circ - T_P^\circ\right) - \frac{\Gamma_w A_w}{(\delta x)_w}\left(T_P^\circ - T_W^\circ\right)\right]\Delta t$$

$$\text{III} = \int_t^{t+\Delta t}\left[\Gamma_n A_n \left.\frac{\partial T}{\partial y}\right|_n - \Gamma_s A_s \left.\frac{\partial T}{\partial y}\right|_s\right] dt$$

$$\left.\frac{\partial T}{\partial y}\right|_n = \frac{(T_N - T_P)}{(\delta y)_n}$$

$$\left.\frac{\partial T}{\partial y}\right|_s = \frac{(T_P - T_S)}{(\delta y)_s}$$

piecewise linear profile assumed

$$\text{III} = \int_t^{t+\Delta t}\left[\frac{\Gamma_n A_n}{(\delta y)_n}\left(T_N - T_P\right) - \frac{\Gamma_s A_s}{(\delta y)_s}\left(T_P - T_S\right)\right] dt$$

Using Fully Explicit again

$$\text{III} = \left[\frac{\Gamma_n A_n}{(\delta y)_n}\left(T_N^\circ - T_P^\circ\right) - \frac{\Gamma_s A_s}{(\delta y)_s}\left(T_P^\circ - T_S^\circ\right)\right]\Delta t$$

Assembling

$$\rho_P (Vol)_P \left(T_P^n - T_P^\circ\right) = \left[\frac{\Gamma_e A_e}{(\delta x)_e}\left(T_E^\circ - T_P^\circ\right) - \frac{\Gamma_w A_w}{(\delta x)_w}\left(T_P^\circ - T_W^\circ\right)\right]\Delta t$$
$$+ \left[\frac{\Gamma_n A_n}{(\delta y)_n}\left(T_N^\circ - T_P^\circ\right. - \frac{\Gamma_s A_s}{(\delta y)_s}\left(T_P^\circ - T_S^\circ\right)\right]\Delta t$$

CONTINUED

5.12 (CONTINUED)

DIVIDE BY Δt AND GATHER TERMS

$$\frac{\rho_p (Vol)_p}{\Delta t}\left(T_p^n - T_p^o\right) = \frac{\Gamma_e A_e}{(\delta x)_e} T_E^o + \frac{\Gamma_w A_w}{(\delta x)_w} T_W^o + \frac{\Gamma_n A_n}{(\delta y)_n} T_N^o + \frac{\Gamma_s A_s}{(\delta y)_s} T_S^o$$

$$- \left(\frac{\Gamma_e A_e}{(\delta x)_e} + \frac{\Gamma_w A_w}{(\delta x)_w} + \frac{\Gamma_n A_n}{(\delta y)_n} + \frac{\Gamma_s A_s}{(\delta y)_s}\right) T_p^o$$

letting $\frac{\Gamma_e A_e}{(\delta x)_e} = D_e \quad \frac{\Gamma_w A_w}{(\delta x)_w} = D_w \quad \frac{\Gamma_n A_n}{(\delta y)_n} = D_n \quad \frac{\Gamma_s A_s}{(\delta y)_s} = D_s$

$$\left[\frac{\rho_p (Vol)_p}{\Delta t}\right] T_p^n = \emptyset\, T_W^n + \emptyset\, T_E^n + \emptyset\, T_N^n + \emptyset\, T_S^n$$

$$+ \left[D_e T_E^o + D_w T_W^o + D_n T_N^o + D_s T_S^o \right]$$

$$+ \left[-\left(D_e + D_w + D_n + D_s\right) + \frac{\rho_p (Vol)_p}{\Delta t}\right] T_p^o$$

$$a_p = \frac{\rho_p (Vol)_p}{\Delta t}$$

$$a_E = a_W = a_N = a_S = 0$$

$$b_p = \left(D_e T_E^o + D_w T_W^o + D_n T_N^o + D_s T_S^o\right] +$$

$$\left[-\left(D_e + D_w + D_n + D_s\right) + \frac{\rho_p (Vol)_p}{\Delta t}\right] T_p^o$$

(b) For the specific problem $D_e = \frac{\Gamma_e A_e}{(\delta x)_e} = \frac{k}{c_p} \frac{\Delta y (1)}{\Delta x}$ $\quad \frac{J}{s} \frac{W}{m K} \frac{kg K}{J} \frac{m\, m}{m}$

$$D_e = \frac{(1.4)}{800} \frac{0.02 (1)}{0.08} = 0.0004375 \left[\frac{kg}{s}\right] \qquad \frac{kg}{s}$$

$$D_w = D_e = 0.0004375 \left[\frac{kg}{s}\right]$$

CONTINUED

$$D_n = \frac{\Gamma_n A_n}{\delta y)_n} = \frac{k}{c_p}\frac{\Delta x(1)}{\Delta y} = \frac{1.4}{800}\frac{0.08}{0.02}(1) = 0.007000 \left(\frac{kg}{s}\right)$$

$$D_s = D_n = 0.007000 \left[\frac{kg}{s}\right]$$

$$\rho_p(Vol)_p = (1400)(0.08)(0.02)(1) = 2.24 \left[kg\right]$$

$$\frac{2.24}{\Delta t} T_p^n = 0.0004375(T_E^\circ + T_W^\circ) + 0.007000(T_N^\circ + T_S^\circ)$$
$$\left[-0.014875 + \frac{2.24}{\Delta t}\right] T_p^\circ$$

(c) For $T_p^\circ = 375 [K]$ $T_W^\circ = 400 [K]$ $T_E^\circ = 450 [K]$ $T_S^\circ = 350 [K]$ $T_N^\circ = 300 [K]$

and $\Delta t = 120 [s]$

$$T_p^n = \left(\frac{120}{2.24}\right)\Big\{ 0.0004375(450+400) + 0.007000(300+350)$$
$$+ \left[-0.014875 + \frac{2.24}{120}\right] 375 \Big\}$$

$$T_p^n = 339.84 [K]$$

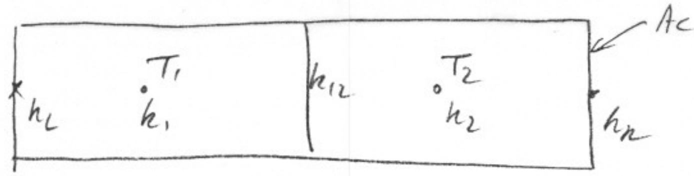
(d) Maximum time step comes from

$$\left(-0.014875 + \frac{2.24}{\Delta t}\right) \geq 0$$

$$-0.014875\,\Delta t + 2.24 \geq 0$$

$$\Delta t \leq \frac{2.24}{0.014875}$$

$$\Delta t \leq 150.6 \text{ seconds}$$



$\dot{Q}''' = 0$

$\Gamma = k$

1D steady conduction

From supplementary sheets for

$$a_P T_P = a_W T_W + a_E T_E + b_P$$

$$a_W = \frac{\Gamma_w A_w}{(\delta x)_w} \qquad a_E = \frac{\Gamma_e A_e}{(\delta x)_e}$$

$$a_P = a_E + a_W$$

$$b_P = 0 \qquad \dot{Q}''' = 0$$

CV. #1 $\quad T_P = T_1 \quad T_W = T_L \quad T_E = T_2$

$$\Gamma_w = k_w = k_L \qquad (\delta x)_w = \Delta x/2 \qquad A_w = A_c$$
$$\Gamma_e = k_e = k_{12} \qquad (\delta x)_e = \Delta x \qquad A_e = A_c$$

$$a_W = \frac{k_L A_c}{\left(\frac{\Delta x}{2}\right)} \qquad a_E = \frac{k_{12} A_c}{\Delta x} \qquad a_P = a_E + a_W$$
$$b_P = 0$$

$$\rightarrow \left[\frac{2k_L A_c}{\Delta x} + \frac{k_{12} A_c}{\Delta x}\right] T_1 = \frac{2k_L A_c}{\Delta x} T_L + \frac{k_{12} A_c}{\Delta x} T_2$$

CV #2

$$\Gamma_w = k_w = k_{12} \qquad (\delta x)_w = \Delta x \qquad A_w = A_c$$
$$\Gamma_e = k_e = k_R \qquad (\delta x)_e = \Delta x/2 \qquad A_e = A_c$$

$$a_W = \frac{k_{12} A_c}{\Delta x} \qquad a_E = \frac{k_R A_c}{\frac{\Delta x}{2}} \qquad a_P = a_E + a_W$$
$$b_P = 0$$

CONTINUED

$$\Rightarrow \left(\frac{k_{12}A_c}{\Delta x} + \frac{2k_R A_c}{\Delta x}\right)T_2 = \frac{k_{12}A_c}{\Delta x}T_1 + \frac{2k_R A_c}{\Delta x}T_R$$

re-arrange & factor out $\left(\frac{A_c}{\Delta x}\right)$ from all terms

$$\rightarrow (2k_L + k_{12})T_1 \quad - k_{12}T_2 = 2k_L T_L \quad (1)$$

$$-k_{12}T_1 \quad + (2k_R + k_{12})T_2 = 2k_R T_R \quad (2)$$

as on the test paper.

b) $k = \text{constant} = 18.7\ (W/mK)$ $\qquad T_L = 100\ [K]$
$A_c = 0.05\ (m^2)$ $\qquad T_R = 900\ [K]$
$\Delta X = 1.87/2 = 0.935\ [m]$

~~E~~quations (1) & (2) become

$$3k\,T_1 - k\,T_2 = 2kT_L$$

$$-k\,T_1 + 3k\,T_2 = 2kT_R$$

divide by k and substitute for T_L & T_R

$$\rightarrow \quad 3T_1 - T_2 = 200 \quad (3)$$

$$-T_1 + 3T_2 = 1800 \quad (4)$$

$$(4) \Rightarrow \quad T_1 = 3T_2 - 1800 \quad (5)$$

CONTINUED

Substitute (5) into (3)

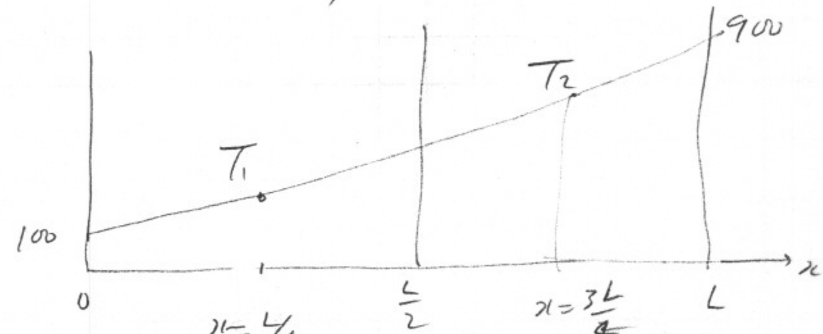
$$3(3T_2 - 1800) - T_2 = 200$$
$$8T_2 = 5600$$
$$T_2 = 700 \text{ (K)}$$

From (5) $T_1 = 3(700) - 1800 = 2100 - 1800$

$$T_1 = 300 \text{ (K)}$$

c) Analytical solution 1D steady no source terms

→ linear



$$T(x) = T_L + \frac{(T_R - T_L)}{L} x$$

$$T_1 = T\left(\frac{L}{4}\right) = T_L + \frac{(T_R - T_L)}{L}\frac{L}{4} = T_L + \frac{1}{4}(T_R - T_L)$$

$$T_1 = 100 + \frac{1}{4}(900 - 100) = 100 + \frac{800}{4} = 100 + 200 = 300$$

$$T_2 = T\left(\frac{3L}{4}\right) = T_L + \frac{3}{4}(T_R - T_L) = 100 + \frac{3}{4}(900 - 100)$$

$$T_2 = 100 + \frac{3(800)}{400} = 100 + 600 = 700 \checkmark$$

T_1 & T_2 nodal values are identical to the analytical solution

CONTINUED