d)

$$k_L = k_1 = k(T_1) = C_k T_1^m$$

$$k_R = k_2 = k(T_2) = C_k T_2^m$$

$$k_{12} = \frac{2 k_1 k_2}{(k_1 + k_2)} = \frac{2 C_k T_1^m C_k T_2^m}{(C_k T_1^m + C_k T_2^m)}$$

$$k_{12} = \frac{2 C_k T_1^m T_2^m}{(T_1^m + T_2^m)}$$

Substitute into T_1 & T_2 equations from part (a)

Eqn (8)

$$\left(2 C_k T_1^m + \frac{2 C_k T_1^m T_2^m}{(T_1^m + T_2^m)}\right) T_1 - \frac{2 C_k T_1^m T_2^m}{(T_1^m + T_2^m)} = 2 C_k T_1^m T_L$$

DIVIDE THROUGH BY $2 C_k$

$$\left[T_1^m + \frac{T_1^m T_2^m}{(T_1^m + T_2^m)}\right] T_1 - \left(\frac{T_1^m T_2^m}{(T_1^m + T_2^m)}\right) T_2 = T_1^m T_L$$

Eqn (9)

$$-\left[\frac{2 C_k T_1^m T_2^m}{(T_1^m + T_2^m)}\right] T_1 + \left[2 C_k T_2^m + \frac{2 C_k T_1^m T_2^m}{(T_1^m + T_2^m)}\right] T_2 = 2 C_k T_2^m T_R$$

$$-\left(\frac{T_1^m T_2^m}{(T_1^m + T_2^m)}\right) T_1 + \left[T_2^m + \frac{T_1^m T_2^m}{(T_1^m + T_2^m)}\right] T_2 = T_2^m T_R$$

CONTINUED

$\Rightarrow$ the new equations with T_1^* T_2^* guessed values in the coefficients for T_1 & T_2:

$$\left[(T_1^*)^m + \frac{(T_1^*)^m (T_2^*)^m}{((T_1^*)^m + (T_2^*)^m)}\right] T_1 - \left[\frac{(T_1^*)^m (T_2^*)^m}{((T_1^*)^m + (T_2^*)^m)}\right] T_2 = (T_1^*)^m T_L$$

$$-\left[\frac{(T_1^*)^m (T_2^*)^m}{[(T_1^*)^m + (T_2^*)^m]}\right] T_1 + \left[(T_2^*)^m + \frac{(T_1^*)^m (T_2^*)^m}{[(T_1^*)^m + (T_2^*)^m]}\right] T_2 = (T_2^*)^m T_R$$

(e) Iteration #1 $\quad T_1^* = 300$

$T_2^* = 700$

$$(T_1^*)^m = (300)^{0.441} = 12.371$$

$$(T_2^*)^m = (700)^{0.441} = 17.976$$

$$\frac{(T_1^*)^m (T_2^*)^m}{(T_1^*)^m + (T_2^*)^m} = \frac{(12.371)(17.976)}{(12.371 + 17.976)} = 7.328$$

$\Rightarrow$

$$(12.371 + 7.328)\, T_1 - 7.328\, T_2 = (12.371)\, 100$$

$$-7.328\, T_1 + (17.976 + 7.328)\, T_2 = (17.976)\, 900$$

$$19.699\, T_1 - 7.328\, T_2 = 1237.1 \qquad (e\text{-}1)$$

$$-7.328\, T_1 + 25.304\, T_2 = 16178.4 \qquad (e\text{-}2)$$

Solve (e-1) & (e-2)

$$T_1 = \frac{(1237.1)(25.304) - 16178.4\,(-7.328)}{(19.699)(25.304) - (-7.328)(-7.328)}$$

CONTINUED

$$T_1 = \frac{149,858.9}{444.764} = 336.94 \ (K)$$

$$T_2 = \frac{16178.4 + 7.328\,(336.94)}{25.304} = 736.94 \ (K)$$

- Iteration #2

$$(T_1^*)^m = (336.94)^{0.441} = 13.021$$

$$(T_2^*)^m = (736.94)^{0.441} = 18.388$$

$$\frac{(T_1^*)^m (T_2^*)^m}{(T_1^*)^m + (T_2^*)^m} = \frac{(13.021)(18.388)}{13.021 + 18.388} = 7.623$$

equations become

$$(13.021 + 7.623)\,T_1 - 7.623\,T_2 = (13.021)\,100$$

$$-7.623\,T_1 + (18.388 + 7.623)\,T_2 = (18.388)\,900$$

$$20.644\,T_1 - 7.623\,T_2 = 1302.1 \qquad (e-3)$$

$$-7.623\,T_1 + 26.011\,T_2 = 16549.2 \qquad (e-4)$$

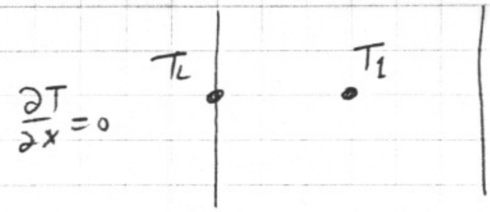
solve (e-3) & (e-4)

$$T_1 = \frac{(1302.1)(26.011) - (16549.2)(7.623)}{(20.644)(26.011) - (-7.623)(-7.623)} = \frac{160,023.5}{478.86}$$

$$T_1 = 334.18 \ (K)$$

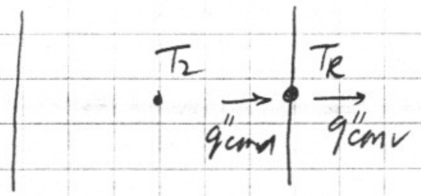
$$T_2 = \frac{16549.2 + 7.623\,(334.18)}{26.011} = 734.18 \ (K)$$

(a)



$$\frac{T_1 - T_L}{\left(\frac{\Delta x}{2}\right)} = 0 \Rightarrow T_1 - T_L = 0 \Rightarrow \boxed{T_L = T_1}$$

(b)



energy balance at right face

$$-k\frac{(T_R - T_2)}{\left(\frac{\Delta x}{2}\right)} = h_\infty (T_R - T_\infty)$$

$$-T_R + T_2 = \frac{1}{2}\left(\frac{h_\infty \Delta x}{k}\right)(T_R - T_\infty)$$

let $\frac{h_\infty \Delta x}{k} = Bi$

$$-2T_R + 2T_2 = Bi\, T_R - Bi\, T_\infty$$

$$(2 + Bi)\, T_R = 2T_2 + Bi\, T_\infty$$

$$T_R = \frac{2}{(2+Bi)} T_2 + \frac{Bi}{(2+Bi)} T_\infty \quad \checkmark$$

(c) Fully Implicit. (Source term is zero)

start from Equations (12.13 to 12.15) with $f_t = 1$

$$a_p T_p^n = a_W T_W^n + a_E T_E^n + b_p$$

$$a_E = \frac{\Gamma_e A_e}{(\delta x)_e} \qquad a_W = \frac{\Gamma_w A_w}{(\delta x)_w}$$

$$b_p = \frac{M_p}{\delta t} T_p^0$$

$$a_p = \frac{M_p}{\delta t} + \frac{\Gamma_e A_e}{(\delta x)_e} + \frac{\Gamma_w A_w}{(\delta x)_w}$$

CONTINUED

5.14 (CONTINUED)

• C.V. for T_1 $\Gamma_e = \Gamma_w = \dfrac{k}{c_p}$ $(\delta x)_w = \dfrac{\Delta x}{2}$ $(\delta x)_e = \Delta x$

$A = 1$ $M_p = \rho\,(Vol) = \rho\,A\,\Delta x = \rho\,\Delta x$

$T_P = T_1$ $T_W = T_L$ $T_E = T_2$

$$\left(\frac{\rho\Delta x}{\Delta t} + \frac{k}{c_p\Delta x} + \frac{2k}{c_p\Delta x}\right)T_1^n = \frac{2k}{c_p\Delta x}T_L^n + \frac{k}{c_p\Delta x}T_2^n + \frac{\rho\Delta x}{\Delta t}T_1^o$$

absorb bc $T_L^n = T_1^n$

$$\left(\frac{\rho\Delta x}{\Delta t} + \frac{3k}{c_p\Delta x}\right)T_1^n = \frac{2k}{c_p\Delta x}T_1^n + \frac{k}{c_p\Delta x}T_2^n + \frac{\rho\Delta x}{\Delta t}T_1^o$$

$$\boxed{\left(\frac{\rho\Delta x}{\Delta t} + \frac{k}{c_p\Delta x}\right)T_1^n = \frac{k}{c_p\Delta x}T_2^n + \frac{\rho\Delta x}{\Delta t}T_1^o}$$

substitute values $\dfrac{\rho\Delta x}{\Delta t} = \dfrac{7800\,(0.015)}{30} = 3.9$

$\dfrac{k}{c_p\Delta x} = \dfrac{58.5}{(390)\,(0.015)} = 10$

$$(10 + 3.9)\,T_1^n = 10\,T_2^n + 3.9\,T_1^o$$

$$\boxed{13.9\,T_1^n = 10\,T_2^n + 3.9\,T_1^o}$$ as in text paper

CV for T_2 $\Gamma_e = \Gamma_w = \dfrac{k}{c_p}$ $(\delta x)_w = \Delta x$ $(\delta x)_e = \dfrac{\Delta x}{2}$ $A = 1$

$T_P = T_2$ $T_W = T_1$ $T_E = T_R$

$$\left(\frac{\rho\Delta x}{\Delta t} + \frac{k}{c_p\Delta x} + \frac{2k}{c_p\Delta x}\right)T_2^n = \frac{k}{c_p\Delta x}T_1^n + \frac{2k}{c_p\Delta x}T_R^n + \frac{\rho\Delta x}{\Delta t}T_2^o$$

absorb the b.c. $T_R^n = \dfrac{2}{(2+Bi)}T_2^n + \dfrac{Bi}{(2+Bi)}T_\infty$

$$\left(\frac{\rho\Delta x}{\Delta t} + \frac{3k}{c_p\Delta x}\right)T_2^n = \frac{k}{c_p\Delta x}T_1^n + \frac{2k}{c_p\Delta x}\left\{\frac{2}{(2+Bi)}T_2^n + \frac{Bi}{(2+Bi)}T_\infty\right\} + \frac{\rho\Delta x}{\Delta t}T_2^o$$

$$\boxed{\left[\frac{\rho\Delta x}{\Delta t} + \frac{3k}{c_p\Delta x} - \frac{2k}{c_p\Delta x}\frac{2}{(2+Bi)}\right]T_2^n = \frac{k}{c_p\Delta x}T_1^n + \frac{k}{c_p\Delta x}\frac{Bi}{(2+Bi)}T_\infty + \frac{\rho\Delta x}{\Delta t}T_2^o}$$

CONTINUED

$$Bi = \frac{h_\infty \Delta x}{k} = \frac{500\,(0.015)}{585} = 0.128205$$

$$\left[3.9 + 30 - 20\left(\frac{2}{2.128205}\right)\right] T_2^{\,n} = 10\,T_1^{\,n} + 20\left(\frac{0.128205}{2.128205}\right)333 + 3.9\,T_2^{\,o}$$

$$\boxed{15.105\,T_2^{\,n} = 10\,T_1^{\,n} + 401.20 + 3.9\,T_2^{\,o}}$$

(d). time step 1

$$T_2^{\,n} = \frac{[39\,(253) + 54.210\,(253) + 5576.7]}{109.96} = 265.18\,[K]$$

$$T_1^{\,n} = \frac{1}{13.9}\left[10\,(265.18) + 3.9\,(253)\right] = 261.76\,(K)$$

- time step 2

$$T_2^{\,n} = \frac{1}{109.96}\left[39\,(261.76) + 54.210\,(265.18) + 5576.7\right] = 274.29\,(K$$

$$T_1^{\,n} = \frac{[10\,(274.29) + 3.9\,(261.76)]}{13.9} = 270.77\,[K]$$

- time step 3

$$T_2^{\,n} = \frac{1}{109.96}\left[39\,(270.77) + 54.210\,(274.29) + 5576.7\right] = 281.98\,[K$$

$$T_1^{\,n} = \frac{[10\,(281.98) + 3.9\,(270.77)]}{13.9} = 278.83\,(K)$$

(e) Fully Explicit (source term is zero)

Start from Equations (12.13) to (12.15) with $f_t = 0$.

$$a_p T_p^{\,n} = a_E T_E^{\,n} + a_W T_W^{\,n} + b_p$$

$$a_E = 0 \qquad a_W = 0$$

$$b_p = \frac{\Gamma_e A_e}{(\delta x)_e} T_E^{\,o} + \frac{\Gamma_w A_w}{(\delta x)_w} T_W^{\,o} + \left[\frac{M_p}{\Delta t} - \left(\frac{\Gamma_e A_e}{(\delta x)_e} + \frac{\Gamma_w A_w}{(\delta x)_w}\right)\right] T_p^{\,o}$$

$$a_p = \frac{M_p}{\Delta t}$$

CONTINUED

• C.V. for T_1 $\quad T_P = T_1 \quad T_W = T_L \quad T_E = T_2$

$$\boxed{\left(\frac{\rho \Delta x}{\Delta t}\right) T_1^n = \frac{k}{c_p \Delta x} T_2^o + \frac{2k}{c_p \Delta x} T_L^o + \left[\frac{\rho \Delta x}{\Delta t} - \left(\frac{k}{c_p \Delta x} + \frac{2k}{c_p \Delta x}\right)\right] T_1^o}$$

$$3.9\, T_1^n = 10\, T_2^o + 20\, T_L^o + (3.9 - 30)\, T_1^o$$

$$\boxed{3.9\, T_1^n = 10 T_2^o + 20 T_L^o - 26.1\, T_1^o}$$ ✓ as on test

• C.V for T_2 $\quad T_P = T_1 \quad T_W = T_1 \quad T_E = T_R$

$$\boxed{\left(\frac{\rho \Delta x}{\Delta t}\right) T_2^n = \frac{k}{c_p \Delta x} T_1^o + \frac{2k}{c_p \Delta x} T_R^o + \left[\frac{\rho \Delta x}{\Delta t} - \left(\frac{k}{c_p \Delta x} + \frac{2k}{c_p \Delta x}\right)\right] T_2^o}$$

$$\boxed{3.9\, T_2^n = 10\, T_1^o + 20\, T_R^o - 26.1\, T_2^o}$$ ✓ as on test

(f) · time step 1

$$T_1^n = \frac{1}{3.9}\left(20(253) + 10(253) - 26.1(253)\right) = 253.00\ (K)$$

$$T_2^n = \frac{1}{3.9}\left(10(253) + 20(253) - 26.1(253)\right) = 253.00\ (K)$$

$$T_L^n = 253.00\ (K)$$

$$T_R^n = 0.93976(253) + 20.060 = 257.82\ (K)$$

· time step 2

$$T_1^n = \frac{1}{3.9}\left(20(253) + 10(253) - 26.1(253)\right) = 253.00\ (K)$$

$$T_2^n = \frac{1}{3.9}\left(10(253) + 20(257.82) - 26.1(253)\right) = 277.72\ (K)$$

$$T_L^n = 253.00\ (K)$$

$$T_R^n = 0.93976(277.72) + 20.060 = 281.05\ (K)$$

CONTINUED

· time step 3

$$T_1^n = \frac{1}{3.9}\left(20(253) + 10(277.72) - 26.1(253)\right) = 316.38\ [K]$$

$$T_2^n = \frac{1}{3.9}\left(10(253) + 20(281.05) - 26.1(277.72)\right) = 231.41\ [K]$$

$$T_L^n = 316.38\ [K]$$

$$T_R^n = 0.93976(231.41) + 20.060 = 237.53\ [K]$$

(g) $\Delta t = 90\ [s]$

	Fully Implicit	Fully Explicit
T_1	278.83 [K]	316.38 (K)
T_2	281.98 [K]	231.41 (K)

Fully Explicit result looks physically unrealistic. Check stability criterion

$$\Delta t \le \frac{\rho C_p (\Delta x)^2}{2h}$$

$$\Delta t \le \frac{(7800)(390)(0.015)^2}{2(58.5)}$$

$$\Delta t \le 5.85\ [s]$$

$\Delta t = 30\ (s)$ used is well above the stability limit for Fully Explicit → Fully Explicit results are unstable.

$$a \frac{d^2T}{dx^2} + S = 0$$

$$\underbrace{a\int_{\forall} \frac{d}{dx}\left(\frac{dT}{dx}\right) d\forall}_{\text{I}} + \underbrace{\int_{\forall} S \, d\forall}_{\text{II}} = 0$$

$$\text{I} = a\int_A \int_w^e \frac{d}{dx}\left(\frac{dT}{dx}\right) dx\, dA = aA\left\{\frac{dT}{dx}\bigg|_e - \frac{dT}{dx}\bigg|_w\right\}$$

(for constant area)

Use linear piecewise profile for $\frac{dT}{dx}$ terms

$$\text{I} = aA\left\{\frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}\right\} = \frac{aA}{\Delta x}\left\{T_E + T_W - 2T_P\right\}$$

$$\text{II} = \bar{S} A(\Delta x)$$

$$\frac{aA}{\Delta x}\left(T_E + T_W - 2T_P\right) + \bar{S} A \Delta x = 0$$

$$\left(\frac{2aA}{\Delta x}\right) T_P = \frac{aA}{\Delta x} T_E + \frac{aA}{\Delta x} T_W + \bar{S} A \Delta x$$

$$\boxed{\begin{array}{ll} a_E = \frac{aA}{\Delta x} & a_W = \frac{aA}{\Delta x} \\ a_P = a_E + a_W & \\ b_P = \bar{S} A \Delta x & \end{array}}$$

CONTINUED

(b) $S = b\left(T_\infty^4 - T^4\right)$

Newton-Raphson linearization:

$$S = S^* + \left.\frac{\partial S}{\partial T}\right|_{T^*}(T - T^*)$$

$$\frac{\partial S}{\partial T} = -4bT^3 \Rightarrow \boxed{\left.\frac{\partial S}{\partial T}\right|_{T^*} = -4b(T^*)^3}$$

$$S^* = S\Big|_{T^*} = b\left(T_\infty^4 - (T^*)^4\right)$$

$$S = bT_\infty^4 - b(T^*)^4 + \left(-4b(T^*)^3\right)(T - T^*)$$

$$= bT_\infty^4 - b(T^*)^4 - 4b(T^*)^3 T + 4b(T^*)^4$$

$$= \left[b\left(T_\infty^4 + 3(T^*)^4\right)\right] + \left[-4b(T^*)^3\right]T$$

for $S = S_c + S_p T_p$

using $T = T_p$ (step-wise profile assumption)

$$\boxed{S_c = b\left(T_\infty^4 + 3(T^*)^4\right)}$$

$$\boxed{S_p = -4b(T^*)^3}$$

Back to the original Eqn

$$\underbrace{\int_V a\frac{d^2T}{dx^2}\,dV}_{\substack{\text{I} \\ \text{already done}}} + \underbrace{\int_V (S_c + S_p T_p)\,dV}_{\substack{\text{II} \\ \text{new form}}}$$

[step-wise for T].

CONTINUED

$$II = \int_A \int_w^e (S_c + S_p T_p)\, dx\, dA = S_c A(\Delta x) + S_p A(\Delta x) T_p$$

Using result for Term I from before

$$\frac{aA}{\Delta x}\left(T_E + T_W - 2T_p\right) + S_c A(\Delta x) + S_p A(\Delta x) T_p = 0$$

$$\left(2\frac{aA}{\Delta x} - S_p A \Delta x\right) T_p = \frac{aA}{\Delta x} T_E + \frac{aA}{\Delta x} T_W + S_c A(\Delta x)$$

$$a_E = \frac{aA}{\Delta x} \qquad a_W = \frac{aA}{\Delta x}$$

$$a_p = a_E + a_W - S_p A(\Delta x)$$

$$b_p = S_c A(\Delta x)$$

Substituting results from derivation of S_c, S_p

$$a_p = a_E + a_W + 4b(T^*)^3 A(\Delta x)$$

$$b_p = b\left(T_\infty^4 + 3(T^*)^4\right) A(\Delta x)$$

$$a_E = \frac{aA}{\Delta x} \qquad a_W = \frac{aA}{\Delta x}$$

$$\underbrace{\int_{\forall} a \frac{d}{dx}\left(\frac{dT}{dx}\right) d\forall}_{I} + \underbrace{\int_{\forall} S \, d\forall}_{II} = 0$$

$$I = a \int_A \int_w^e \frac{d}{dx}\left(\frac{dT}{dx}\right) dx\, dA = aA\left(\left.\frac{dT}{dx}\right|_e - \left.\frac{dT}{dx}\right|_w\right)$$

(for constant area)

- use linear temperature profile for $\frac{dT}{dx}$ terms

$$I = aA\left\{\frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}\right\} = \frac{aA}{\Delta x}\left\{T_E + T_W - 2T_P\right\}$$

$$II = \bar{S} A \Delta x$$

(for constant source term $\bar{S}$)

$$\frac{aA}{\Delta x}\left(T_E + T_W - 2T_P\right) + \bar{S} A \Delta x = 0$$

$$\rightarrow \quad \left(\frac{2aA}{\Delta x}\right) T_P = \left(\frac{aA}{\Delta x}\right) T_E + \left(\frac{aA}{\Delta x}\right) T_W + \left(\bar{S} A \Delta x\right)$$

So

$$\boxed{\begin{aligned} a_E &= \frac{aA}{\Delta x} \qquad a_W = \frac{aA}{\Delta x} \\ a_P &= a_E + a_W \\ b_P &= \bar{S} A \Delta x \end{aligned}}$$

CONTINUED

(b) when

$$S(T) = -b(T - T_\infty)^3$$

We want to derive coefficients for

$$S = S_c + S_p T_p$$

using a Newton-Raphson linearization

$$S = S^* + \left.\frac{\partial S}{\partial T}\right|_{T_p^*} (T_p - T_p^*)$$

$$\frac{\partial S}{\partial T} = -b\,3\,(T - T_\infty)^2$$

$$S = -b(T_p^* - T_\infty)^3 + \left(-3b(T_p^* - T_\infty)^2\right)(T_p - T_p^*)$$

$$S = \left[-b(T_p^* - T_\infty)^3 + 3b(T_p^* - T_\infty)^2 T_p^*\right] + \left[-3b(T_p^* - T_\infty)^2\right] T_p$$

$$\boxed{\begin{aligned} S_c &= -b(T_p^* - T_\infty)^3 + 3b(T_p^* - T_\infty)^2 T_p^* \\ S_p &= -3b(T_p^* - T_\infty)^2 \end{aligned}}$$

CONTINUED

Redo the source term

$$\int_{\forall} S \, d\forall = S_c (A \Delta x) + S_p (A \Delta x) T_p$$

gather terms again

$$\frac{aA}{\Delta x}\left(T_E + T_W - 2T_p\right) + S_c (A \Delta x) + S_p (A \Delta x) T_p = 0$$

$$\left[2\frac{aA}{\Delta x} - S_p (A \Delta x)\right] T_p = \left(\frac{aA}{\Delta x}\right) T_E + \left(\frac{aA}{\Delta x}\right) T_W + S_c A (\Delta x)$$

$$\boxed{\begin{aligned} a_p &= a_E + a_W + 3bA\Delta x \left(T_p^* - T_\infty\right)^2 \\ a_E &= \frac{aA}{\Delta x} \qquad\qquad a_W = \frac{aA}{\Delta x} \\ b_p &= \left[-b\left(T_p^* - T_\infty\right)^3 + 3b\left(T_p^* - T_\infty\right)^2 T_p^*\right] A \Delta x \end{aligned}}$$

(a) $$\int_{V} \frac{d^2T}{dx^2}\,dV + \int_{V} \dot{S}''' \,dV = 0$$

$$\underbrace{\int_{V} \frac{d}{dx}\left(\frac{dT}{dx}\right)dV}_{I} + \underbrace{\int_{V} \left(S_c + S_p T\right)dV}_{II} = 0$$

$$I = A_e \left.\frac{dT}{dx}\right|_e - A_w \left.\frac{dT}{dx}\right|_w = A\frac{(T_E - T_P)}{\Delta x} - A\frac{(T_P - T_W)}{\Delta x}$$

$$I = \frac{A}{\Delta x} T_E + \frac{A}{\Delta x} T_W - \frac{2A}{\Delta x} T_P$$

$$II = S_c (Vol)_P + S_P (Vol)_P T_P = S_c A \Delta x + S_P (A \Delta x) T_P$$

Gather terms

$$\frac{A}{\Delta x} T_E + \frac{A}{\Delta x} T_W - \frac{2A}{\Delta x} T_P + S_c A \Delta x + S_P (A \Delta x) T_P = 0$$

$$\left(\frac{2A}{\Delta x} - S_P A \Delta x\right) T_P = \frac{A}{\Delta x} T_E + \frac{A}{\Delta x} T_W + S_c A \Delta x$$

$$\Rightarrow \boxed{\begin{array}{ll} a_E = \dfrac{A}{\Delta x} & a_W = \dfrac{A}{\Delta x} \\[2ex] a_P = \dfrac{2A}{\Delta x} - S_P A \Delta x & b_P = S_c A \Delta x \end{array}}$$

CONTINUED

5.17 (CONTINUED)

(b) $$S = -c\,(T - T_\infty)^{1/4}$$

$$\frac{\partial S}{\partial T} = -c\,\frac{1}{4}\,(T - T_\infty)^{-3/4}(1)$$

$$(\dot{S}''')_p = -c\,(T_p^* - T_\infty)^{1/4} + -\frac{c}{4}(T_p^* - T_\infty)^{-3/4}(T_p - T_p^*)$$

$$(\dot{S}''')_p = -c\,(T_p^* - T_\infty)^{1/4} + \frac{c}{4}(T_p^* - T_\infty)^{-3/4}\,T_p^* - \frac{c}{4}(T_p^* - T_\infty)^{-3/4}\,T_p$$

$$(\dot{S}''')_p = \left[-c\,(T_p^* - T_\infty)^{1/4} + \frac{c}{4}(T_p^* - T_\infty)^{-3/4}\,T_p^*\right] + \left[-\frac{c}{4}(T_p^* - T_\infty)^{-3/4}\right] T_p$$

$$\therefore \quad S_c = -c(T_p^* - T_\infty)^{1/4} + \frac{c}{4}(T_p^* - T_\infty)^{-3/4}\,T_p^*$$

$$S_p = -\frac{c}{4}(T_p^* - T_\infty)^{-3/4}$$

5.18

(a) $\dot{S}''' = -C(T-T_\infty)^{1/4}$

$$\frac{\partial \dot{S}'''}{\partial T} = -C\left(\frac{1}{4}\right)(T-T_\infty)^{-3/4}(1) = -\frac{C}{4}(T-T_\infty)^{-3/4}$$

$$(\dot{S}''')_p = -C(T_p^*-T_\infty)^{1/4} + \left\{-\frac{C}{4}(T_p^*-T_\infty)^{-3/4}\right\}(T_p - T_p^*)$$

$$= -C(T_p^*-T_\infty)^{1/4} + \frac{C}{4}(T_p^*-T_\infty)^{-3/4}\,T_p^*$$

$$- \frac{C}{4}(T_p^*-T_\infty)^{-3/4}\,T_p$$

$$(\dot{S}''')_p = \left[-C(T_p^*-T_\infty)^{1/4} + \frac{C}{4}T_p^*(T_p^*-T_\infty)^{-3/4}\right]$$

$$+ \left[-\frac{C}{4}(T_p^*-T_\infty)^{-3/4}\right]T_p$$

$$S_C = -C(T_p^*-T_\infty)^{1/4} + \frac{C}{4}T_p^*(T_p'-T_\infty)^{-3/4}$$

$$S_P = -\frac{C}{4}(T_p^*-T_\infty)^{-3/4}$$

(b) $\dot{S}''' = -K(T^4 - T_{sur}^4)$

$$\frac{\partial \dot{S}'''}{\partial T} = -K\,4T^3 = -4KT^3$$

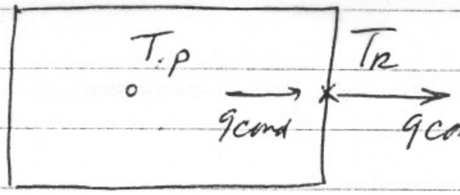
$$(\dot{S}''')_p = -K\left((T_p^*)^4 - T_{sur}^4\right) + \left\{-4K(T_p^*)^3\right\}(T_p - T_p^*)$$

$$(\dot{S}''')_p = -K(T_p^*)^4 + KT_{sur}^4 + 4K(T_p^*)^4 - 4K(T_p^*)^3\,T_p$$

$$(\dot{S}''')_p = \left[3K(T_p^*)^4 + KT_{sur}^4\right] + \left[-4K(T_p^*)^3\right]T_p$$

$$S_C = 3K(T_p^*)^4 + K\,T_{sur}^4$$

$$S_P = -4K(T_p^*)^3$$



Energy balance at the right boundary

$$q_{conv} = q_{cond}$$

$$h_\infty A_c (T_R - T_\infty) = -k_R A_c \left.\frac{dT}{dx}\right|_R = -k_R A_c \frac{(T_R - T_P)}{\left(\frac{\Delta x}{2}\right)}$$

$$h_\infty (T_R - T_\infty) = \frac{-2k}{\Delta x}(T_R - T_P)$$

$$h_\infty = C_1 (T_R - T_\infty)^m$$

$$\Rightarrow \quad C_1 (T_R - T_\infty)^m (T_R - T_\infty) = -\frac{2k}{\Delta x}(T_R - T_P)$$

$$\boxed{C_1 (T_R - T_\infty)^{m+1}} = \frac{-2k}{\Delta x}(T_R - T_P) \qquad (1)$$

$q''_{conv.}$ non-linear term (must be linear in T_R)

Newton-Raphson on q''_{conv} (or q_{conv} since A_c = constant)

$$q_{conv} = (q_{conv})^* + \left(\frac{\partial q_{conv}}{\partial T_R}\right)^* (T_R - T_R^*)$$

$$(q_{conv})^* = A_c C_1 \cdot (T_R^* - T_\infty)^{m+1}$$

CONTINUED

5.19 (CONTINUED)

$$\frac{\partial q_{conv}}{\partial T_R} = \frac{\partial}{\partial T_R}\left(A_c C_1 (T_R - T_\infty)^{m+1} \right)$$

$$= A_c C_1 (m+1) (T_R - T_\infty)^m \quad (1)$$

$$\left(\frac{\partial q_{conv}}{\partial T_R}\right)^* = A_c C_1 (m+1) (T_R^* - T_\infty)^m$$

$$\Rightarrow q_{conv} = A_c C_1 (T_R^* - T_\infty)^{m+1} + A_c C_1 (m+1)(T_R^* - T_\infty)^m (T_R - T_R^*)$$

$$q_{conv} = A_c C_1 (T_R^* - T_\infty)^{m+1} - A_c C_1 (m+1) (T_R^* - T_\infty)^m T_R^*$$
$$+ A_c C_1 (m+1) (T_R^* - T_\infty)^m T_R$$

$$q''_{conv} = \frac{q_{conv}}{A_c} = \left[C_1 (T_R^* - T_\infty)^{m+1} - C_1 (m+1) (T_R^* - T_\infty)^m T_R^* \right]$$
$$+ \left[C_1 (m+1) (T_R^* - T_\infty)^m \right] T_R \quad (2)$$

Substitute (2) in for q''_{conv} in (1)

$$\left[C_1 (T_R^* - T_\infty)^{m+1} - C_1 (m+1) (T_R^* - T_\infty)^m T_R^* \right] + \left[C_1 (m+1) (T_R^* - T_\infty)^m \right] T_R$$

$$= -\frac{2k}{\Delta x} (T_R - T_p)$$

CONTINUED

$$\left[C_1(m+1)(T_R^* - T_\infty)^m + \frac{2k}{\Delta x}\right] T_R = \left(\frac{2k}{\Delta x}\right) T_P$$

$$- \left[C_1 (T_R^* - T_\infty)^{m+1} - C_1 (m+1) (T_R^* - T_\infty)^m T_R^*\right]$$

$$\boxed{\begin{aligned} a_{PR} &= C_1 (m+1) (T_R^* - T_\infty)^m + \frac{2k}{\Delta x} \\ a_{WR} &= \frac{2k}{\Delta x} \\ b_{PR} &= - \left[C_1 (T_R^* - T_\infty)^{m+1} - C_1 (m+1) (T_R^* - T_\infty)^m T_R^*\right] \end{aligned}}$$