Problems

4.1 Given the following equation set

$$\begin{aligned} 2.0\phi_1 \quad -\phi_2 &= 10.0 \\ -\phi_1 \quad +6.0\phi_2 &= 6.0 \end{aligned}$$

(a) Solve for ϕ_1 and ϕ_2 simultaneously.

(b) Solve for ϕ_1 and ϕ_2 by iteration. Use 0.0 as your initial guess for both ϕ_1 and ϕ_2 and show your results for the first 4 iterations.

4.2 Given the following equation set

$$\begin{aligned} 3\,\phi_1 \quad -\,\phi_2 &= 14 \\ -\,\phi_1 \quad +\,2\,\phi_2 &= -3 \end{aligned}$$

(a) Solve for ϕ_1 and ϕ_2 simultaneously.

(b) Solve for ϕ_1 and ϕ_2 using Gauss-Seidel iteration. Use 0.0 as your initial guess for both ϕ_1 and ϕ_2 and show your results for the first 4 iterations.

4.3 Given the following equation set

$$\begin{aligned} 2\,\phi_1 \quad -\,\phi_2 &= 14 \\ -3\,\phi_1 \quad +\,4\,\phi_2 &= -46 \end{aligned}$$

(a) Solve for ϕ_1 and ϕ_2 using Gauss-Seidel iteration. Use 2.0 as your initial guess for both ϕ_1 and ϕ_2 and show your results for the first 5 iterations.

(b) Calculate the magnitude of the percentage change between the 5th and 4th iteration values from part (a) for each of ϕ_1 and ϕ_2.

(c) Solve for ϕ_1 and ϕ_2 simultaneously.

4.4 Given the following equation set

$$\begin{aligned} 2\,\phi_1 \quad +\,\phi_2 &= 17 \\ 3.5\,\phi_1 \quad +\,2\,\phi_2 &= 28 \end{aligned}$$

(a) Solve for ϕ_1 and ϕ_2 graphically. Use quadrule paper for your graph. Clearly label the graph.

(b) Solve for ϕ_1 and ϕ_2 by elimination of unknowns.

(c) Check your answer from part (b) by substituting it back into the original equations.

4.5 Given the following system of equations:

$$\begin{array}{rrrcr} x_1 & -\ 3\,x_2 & +\ 12\,x_3 & = & 269 \\ 5\,x_1 & -\ 13\,x_2 & +\ 2\,x_3 & = & 379 \\ x_1 & -\ 14\,x_2 & & = & -\ 45 \end{array}$$

(a) Solve the system of equations using Gauss Elimination.

(b) Solve the system of equations using the Gauss-Jordan method.

(c) Solve the system of equations using the Gauss-Seidel method. Use an initial guess of $x_1 = x_2 = x_3 = 0$ and stop iteration using $\epsilon_s = 1\%$.

4.6 Given the following system of equations:

$$\begin{array}{rrrcr} x_1 & -\ 12\,x_2 & +\ 5\,x_3 & = & 16 \\ 3\,x_1 & & -\ 2\,x_3 & = & 272 \\ x_1 & -\ 3\,x_2 & +\ 10\,x_3 & = & 146 \end{array}$$

(a) Solve the system of equations using Gauss Elimination.

(b) Check your answer from part (a) by substituting it into the original equations.

(c) Solve the system of equations using the Gauss-Seidel method. Use an initial guess of $x_1 = x_2 = x_3 = 0$ and stop iteration using $\epsilon_s = 1\%$. Throughout your calculations, keep 5 significant figures.

4.7 Given the following system of equations:

$$\begin{array}{rrrrcr} 3\,x_1 & -\ 2\,x_2 & & & = & 67 \\ -\ x_1 & +\ 2\,x_2 & -\ x_3 & & = & -25 \\ & -\ x_2 & +\ 2\,x_3 & -\ x_4 & = & 10 \\ & & -\ 2\,x_3 & +\ 3\,x_4 & = & -10 \end{array}$$

(a) Solve the system of equations using Gauss Elimination.

(b) Set up and start to solve the system of equations using the Gauss-Seidel method. Do only **two iterations** (one iteration being one set of calculations of all four x values). Use an initial guess of $x_1 = x_2 = x_3 = x_4 = 0$. Throughout your calculations, keep 5 significant figures and calculate relative error.

4.8 Given the following set of equations:

$$
\begin{array}{rcrcrcl}
3\,x_1 & + & x_2 & + & 6\,x_3 & = & 2 \\
2\,x_1 & + & x_2 & + & 3\,x_3 & = & 7 \\
x_1 & + & x_2 & + & x_3 & = & 4
\end{array}
$$

(a) Solve the system of equations using Gauss Elimination.

(b) Set up and start to solve the system of equations using the Gauss-Seidel method. Do only **two iterations** (i.e. calculate all x values two times). Use an initial guess of $x_1 = x_2 = x_3 = 0$. Throughout your calculations, keep 5 significant figures and calculate relative error.

4.9 Given the following general three-point equation for T_i:

$$(a_P)_i T_i = (a_W)_i T_{i-1} + (a_E)_i T_{i+1} + (b_P)_i$$

which can also be written as:

$$(-a_W)_i T_{i-1} + (a_P)_i T_i + (-a_E)_i T_{i+1} = (b_P)_i$$

the TDMA uses the following relationship

$$T_i = P_i T_{i+1} + Q_i$$

where

$$P_i = \frac{(a_E)_i}{((a_P)_i - (a_W)_i P_{i-1})}$$

and

$$Q_i = \frac{(a_W)_i Q_{i-1} + (b_P)_i}{((a_P)_i - (a_W)_i P_{i-1})}$$

The starting conditions are: $P_1 = \frac{(a_E)_1}{(a_P)_1}$ and $Q_1 = \frac{(b_P)_1}{(a_P)_1}$.

For the following tri-diagonal matrix equation set:

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 625 \\ 25 \\ 1225 \end{bmatrix}$$

(a) Solve for P_i and Q_i for $i = 1, 2,$ and 3.

(b) Solve the set of equations for T_1, T_2, and T_3 using back substitution.

(c) Check your answer from part (b) by substituting it back into the original equations.

4.10 Given the following general three-point equation for ϕ_i:

$$(a_P)_i \phi_i = (a_W)_i \phi_{i-1} + (a_E)_i \phi_{i+1} + (b_P)_i$$

which can also be written as:

$$(-a_W)_i \phi_{i-1} + (a_P)_i \phi_i + (-a_E)_i \phi_{i+1} = (b_P)_i$$

the TDMA uses the following relationship

$$\phi_i = P_i \phi_{i+1} + Q_i$$

where

$$P_i = \frac{(a_E)_i}{[(a_P)_i - (a_W)_i P_{i-1}]}$$

and

$$Q_i = \frac{(a_W)_i Q_{i-1} + (b_P)_i}{[(a_P)_i - (a_W)_i P_{i-1}]}$$

The starting conditions are: $P_1 = \frac{(a_E)_1}{(a_P)_1}$ and $Q_1 = \frac{(b_P)_1}{(a_P)_1}$.

For the following tri-diagonal matrix equation set:

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{Bmatrix} = \begin{bmatrix} 67 \\ -25 \\ 10 \\ -10 \end{bmatrix}$$

(a) Solve for P_i and Q_i for i = 1, 2, 3, and 4.

(b) Solve the set of equations for ϕ_1, ϕ_2, ϕ_3, and ϕ_4 using back substitution.

(c) Check your answer from part (b) by substituting it back into the original equations.