Problems

7.1 You have run a computer program that solves for the temperature field for a steady one-dimensional convection-diffusion problem with a source term. The program used uniform grid spacing and in part of the solution domain it produced the results shown in Problem Figure 7.1 that you want to check.

The following values were used:

$\rho = 0.5136$	$[kg/m^3]$	$U = 0.01$	$[m/s]$	$A = 1.00$	$[m^2]$
$c_p = 2.2870$	$[kJ/kg \cdot K]$	$\dot{Q}''' = 1.00$	$[kW/m^3]$	$\Delta x = 0.02$	$[m]$
$k = 0.0370$	$[W/m \cdot K]$				

You also know that the computer program used the general convection and diffusion weighting factors α and β. To reduce computational effort, however, the program used the following expressions for α and β as approximations to the exponential differencing scheme:

$$|\alpha| = \frac{\mathrm{Pe}^2}{10 + 2\,\mathrm{Pe}^2}$$

$$\beta = \frac{1 + 0.005\,\mathrm{Pe}^2}{1 + 0.05\,\mathrm{Pe}^2}$$

where α takes the sign of the mass flux.

Use the nodal temperatures and the problem description given above to perform the following calculations for the control volume for T_2 (making sure that you are consistent with the approximations that have been made in the program):

(a) Calculate the values of T at the east and west faces of the control volume.

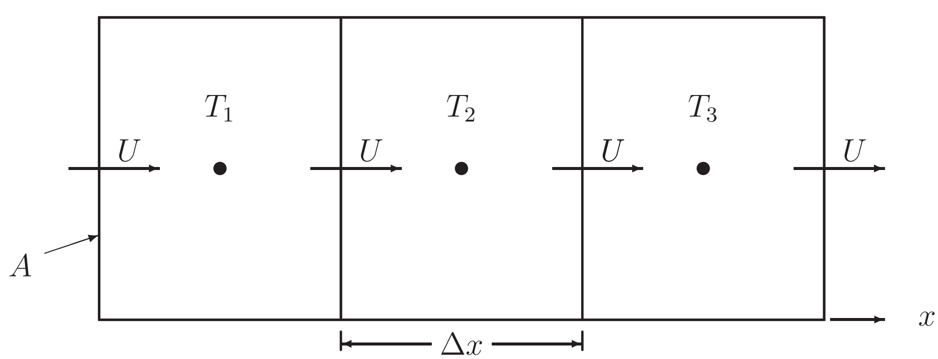
(b) Calculate the values of $\frac{dT}{dx}$ at the east and west faces of the control volume.

(c) Demonstrate that energy is conserved in the control volume.

7.2 You have run a computer program that solves for the temperature field for a steady one-dimensional convection-diffusion problem with a source term. The program used uniform grid spacing and in part of the solution domain it produced the following results shown in Problem Figure 7.2 that you want to check.

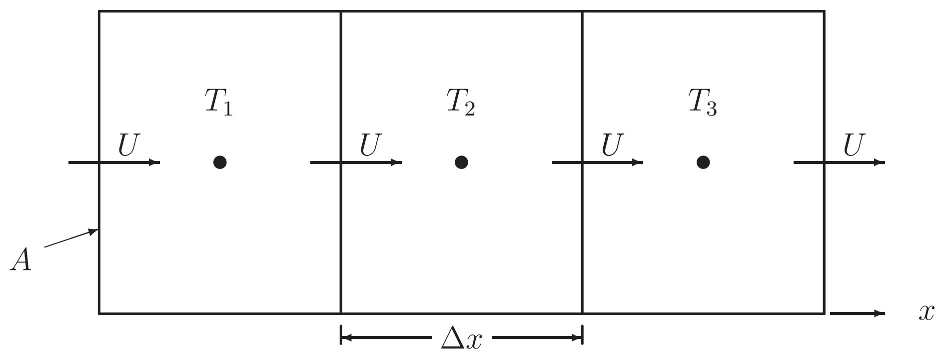
The following values were used:

$\rho = 0.24883$	$[kg/m^3]$	$U = -0.09$	$[m/s]$	$A = 1.00$	$[m^2]$
$c_p = 1.207$	$[kJ/kg \cdot K]$	$\dot{Q}''' = 2.00$	$[kW/m^3]$	$\Delta x = 0.01$	$[m]$
$k = 0.0901$	$[W/m \cdot K]$	$\mu = 5.3\times10^{-5}$	$[N \cdot s/m^2]$		



$T_1 = 405.0334\ [K]$ $\quad T_2 = 406.8096\ [K]$ $\quad T_3 = 418.2758\ [K]$

Problem Figure 7.1: Schematic for Problem 7.1.



$T_1 = 725.0515\ [K] \qquad T_2 = 703.8942\ [K] \qquad T_3 = 702.1379\ [K]$

Problem Figure 7.2: Schematic for Problem 7.2.

You also know that the computer program used the general convection and diffusion weighting factors α and β. In addition, this version of the program used the Exponential Differencing Scheme, so the following expressions for α and β were used:

$$\alpha = \frac{1}{2} - \frac{\left[\exp\left(\frac{1}{2}\mathrm{Pe}\right) - 1\right]}{\left[\exp\left(\mathrm{Pe}\right) - 1\right]} \qquad\qquad \beta = \frac{\mathrm{Pe}\left[\exp\left(\frac{1}{2}\mathrm{Pe}\right)\right]}{\left[\exp\left(\mathrm{Pe}\right) - 1\right]}$$

where the Peclet number is defined by

$$\mathrm{Pe} = \frac{\dot{m}}{D}$$

Use the nodal temperatures and the problem description given above to perform the following calculations for the control volume for T_2 (making sure that you are consistent with the approximations that have been made in the program):

(a) Calculate the values of T at the east and west faces of the control volume.

(b) Calculate the values of $\frac{dT}{dx}$ at the east and west faces of the control volume.

(c) Demonstrate that energy is conserved in the control volume.

7.3 The exact solution for the differential equation for steady one-dimensional convection and diffusion with no source term and boundary conditions $\phi = \phi_L$ at $x = x_L$ and $\phi = \phi_R$ at $x = x_R$ ($x_R > x_L$) is:

$$\frac{(\phi(x) - \phi_L)}{(\phi_R - \phi_L)} = \frac{\left[\exp\left(\mathrm{Pe}\,\frac{(x - x_L)}{(x_R - x_L)}\right) - 1\right]}{\left[\exp\left(\mathrm{Pe}\right) - 1\right]}$$

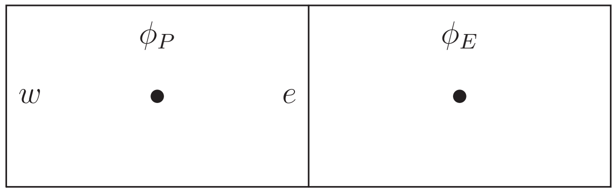
where the Peclet number is defined by

$$\mathrm{Pe} = \frac{\rho U (x_R - x_L)}{\Gamma}$$

(a) From the exact solution given above derive an expression for the weight α_e used in the equation:

$$\phi_e = \left(\frac{1}{2} + \alpha_e\right)\phi_P + \left(\frac{1}{2} - \alpha_e\right)\phi_E$$

where ϕ_e is the value of ϕ at the east face of control volume for ϕ_P as shown in Problem Figure 7.3.



Problem Figure 7.3: Schematic for Problem 7.3.

Use uniform grid spacing.

(b) For the three limits:

i. $\mathrm{Pe} \to 0$

ii. $\mathrm{Pe} \to +\infty$

iii. $\mathrm{Pe} \to -\infty$

use the equation for α_e from part (a) to obtain the values of α_e in each of the limits.

(c) For the three limits in part (b), what is the value of ϕ_e (in terms of the nodal values) ?

7.4 The convection and diffusion weights, α and β, for the Exponential Differencing Scheme (EDS) are given by the expressions below:

$$\alpha = \frac{1}{2} - \frac{\left[\exp\left(\frac{1}{2}\mathrm{Pe}\right) - 1\right]}{\left[\exp\left(\mathrm{Pe}\right) - 1\right]} \qquad \beta = \frac{\mathrm{Pe}\left[\exp\left(\frac{1}{2}\mathrm{Pe}\right)\right]}{\left[\exp\left(\mathrm{Pe}\right) - 1\right]}$$

(a) Use the equation for α given above to obtain the values of α in each of the three limits below.

i. $\mathrm{Pe} \to 0$

ii. $\mathrm{Pe} \to +\infty$

iii. $\mathrm{Pe} \to -\infty$

(b) Use the equation for β given above to obtain the values of β in each of the three limits below.

i. $\mathrm{Pe} \to 0$

ii. $\mathrm{Pe} \to +\infty$

iii. $\mathrm{Pe} \to -\infty$