Amortizing Premium and Discount
(from section 6.2)

Definition. bond is said to **sell at a premium** if the price P is larger than the redemption value; this is equivalent to having the yield rate being smaller than the coupon rate. In this case the difference $P - C$ is called the **premium**.

Definition. bond is said to **sell at a discount** if the price P is smaller than the redemption value; this is equivalent to having the yield rate being larger than the coupon rate. In this case the difference $C - P$ is called the **discount**.

A bond is traded in the market with a prevailing market yield rate. However , for the purpose of accounting, a second method of pricing is used in which the price of the bond is based on the investor's yield rate at the time the bond was purchased. This price is called **book value** and the investor's yield rate is called the **book yield** . The book value at time t immediately after the coupon is paid is denoted by BV_t. There are two ways of calculating the book value: the **prospective formula** and the **retrospective formula**.

Book Value (prospective formula):

The book value immediately after the t-th coupon is paid is:

$BV_t = (\textbf{coupon})\, a_{\overline{n-t}|j} + (\textbf{redemption value})\, v_j^{n-t}$

where:

j=book yield (also called: investor's yield)=yield at the time of purchase

Book Value (retrospective formula):

The book value immediately after the t-th coupon is paid is:

$BV_t = (\textbf{purchase price})(1+j)^t - (\textbf{coupon})\, s_{\overline{t}|j}$

where:

j=book yield (also called: investor's yield)=yield at the time of purchase

Note that:

$$BV_0 = P \qquad\qquad BV_n = C$$

Suppose that two Friends, Friend A and Friend B put some money together and invest that sum. Suppose that each of them puts \$500. Suppose that Friend A is the manager of the fund so established. Suppose that at the end of the first year the fund earns 10%, i.e. it has earned 100 dollars. So, each of them should receive 50 dollars at the end of the first year. But the manager (Friend A) gives only 30 dollars to Friend B. But instead, the manager increase the share of Friend B to 500+20=520 to make up for the remaining 20 dollars. This is the same method we are going to amortize a bond.

Example (Examples 6.6 and 6.7 of the textbook). Suppose that a 5 year bond with semiannual coupons is redeemable at par. Suppose that the bond was purchased to yield 7% annually effective. Suppose that the bond pays interest at $i^{(2)} = 5\%$.

(i) Find the price of the bond per \$100 par value. Is it sold at a discount or a premium?.

(ii) For this bond, construct the amortization schedule.

Solution to part (i). The effective rate per coupon period is:

$i = (1.07)^{0.5} - 1 = 0.03441$

Each coupon is $(100)(\frac{0.05}{2}) = 2.5$. There are 10 coupon payments.

$$P = 2.5a_{\overline{10}|0.03441} + 100(1.03441)^{-10} = 92.15$$

The bond is sold at a discount. This happened because the yield rate per coupon period is larger than the interest rate.

Solution to part (ii). We have invested 92.15 and we are expecting to collect interest every half-year. At the end of the first half-year, our investment should worth (1+j)(92.15)=(1.03441)(92.15). The difference between this value and the original investment of 92.15 is the interest we expect to collect. So, the interest that we expect is $I_1 = (1+j)(92.15) - (92.15) = j(92.15) = (0.03441)(92.15) = 3.17$. But we actually receive 2.50. The textbook calls the shortage of

$$PA_1 = 3.17 - 2.50 = 0.67$$

the **Accumulation of discount**. This value should be added to the book value to adjust it. So, the new book value will be

$$BV_1 = BV_0 + \text{Accumulation of discount} = 92.15 + 0.67 = 92.82$$

So, at the end of the first half-year, after the coupon is paid, our investment is actually worth 92.82.

At the end of second half-year we expect to receive the interest

$$I_2 = j(92.82) = (0.03441)(92.82) = 3.19$$

but we actually receive 2.50. The Accumulation of discount for the second period is:

$$PA_2 = 3.19 - 2.50 = 0.69$$

This should be added to our previous book value to adjust it:

$BV_2 = BV_1 + \text{Accumulation of discount} = 92.82 + 0.69 = 93.52$

and , this process goes on and on until the last book value equal to redemption amount. The table on page 181 has all details. Review the table with the students.

Important Note. In your exam you may be asked to complete two or three three rows of an amortization schedule.

Definition. For any bond (at par, at discount or at premium) the difference

$$PA_t = BV_{t-1} - BV_t$$

is called the **premium amortization** for the period t.

Note. As you may see from the example above, when a bond is purchased at discount, all values of the premium amortization are negative. In this case, the value $-PA_t$ is called the **Accumulation of discount**. When the bond is purchased at premium, the values PA_t are all positive, and they are called **Amortization of premium**.

Note. The total amount of interest is equal the sum of coupons plus the redemption value minus the price paid for. So, for the table on page 181 the total interest is

$(10)(2.5) + 100.00 - 92.15 = 32.85$

It is interesting to note that the sum of the values in the third column called "Interest earned" is equal to this total interest (check it out for yourself).

Note. The adjustment of the book value is used for tax purposes too.

Note. In the study of bonds we have two values "face value" and "redemption value" and we have two rates , the "coupon rate" and "yield rate" . If we multiply the face value by the

coupon rate and also multiply the redemption value by the yield rate and then take the difference, and then multiply it by the annuity $a_{\overline{n}|}$, then we will have $P - C$. Therefore if we add C to it, then we will have the price:

$$P = C + (Fr - Ci)a_{\overline{n}|}$$

where n is the number of coupon periods. Now compare the formula $P - C = (Fr - Ci)a_{\overline{n}|}$ with the following formula for PA_t :

Theorem.

$$PA_t = (Fr - Ci)v^{n-t+1}$$

Show the proof of this from page 178 to the students.

Work on example 6.8 on page 181 and the table on page 182.