Limited Fluctuation Credibility

also called

Classical Credibility

Limited Fluctuation Credibility (also called the **classical approach**):
Update the prediction of loss, as a weighted average of the prediction based on recent data and the rate taken from the insurance manual.

$$\textbf{Limited Fluctuation Credibility}\begin{cases}\textbf{Full Credibility}: \text{ the updated prediction is based on recent data only}\\ \textbf{Partial Credibility}: \text{ the recent data is insufficient for updating prediction}\end{cases}$$

We apply credibility theory to these measures:

- **(i)**: **Claim Frequency** N.
- **(ii)**: **Aggregate Loss** S.
- **(iii)**: **Claim Severity** This refers to the average claim severity: $\frac{S}{N}$.
- **(iv)**: **Pure Premium** If E denotes the number of exposure units, then the quotient $\frac{S}{E}$ is called the pure premium.

Note. The claim frequency N is random, but number of exposure units E is fixed over time (like the number of workers covered for work compensation plan).

From Chapter 8 of MAHLER-DEAN. Pure Premiums are defined as losses divided by exposures. For example, if 200 cars generate claims that total to \$80,000 during a year, then the observed Pure

Premium is $\frac{80,000}{200}$ or \$400 per car-year. Pure premiums are the product of frequency and severity:

$$\begin{aligned}
\textbf{Pure Premiums} &= \frac{\textbf{Losses}}{\textbf{Exposures}} \\
&= \left(\frac{\textbf{Losses}}{\textbf{Number of Claims}}\right)\left(\frac{\textbf{Number of Claims}}{\textbf{Exposures}}\right) \\
&= \textbf{(Severity)(Frequency)}
\end{aligned}$$

Since they depend on both the number of claims and the size of claims, pure premiums have more reasons to vary than do either frequency or severity individually.

In the above example, if there has been 300 claims, then frequency is

$$\frac{300}{400} = 0.75 \ \frac{\text{claim}}{\text{exposure unit}}$$

and the severity is

$$\frac{80,000}{300} = 266.67 \ \frac{\$}{\text{claim}}$$

Note. If the severity and frequency are independent , then we can use the following formula to calculate the vairance of the pure premium:

$$\sigma_{pp}^2 = \mu_f \, \sigma_X^2 + \mu_X^2 \, \sigma_f^2 \qquad\qquad X \text{ being the severity}$$

Note. If the predicted loss value based on the company's manual is denoted by M, and the predicted value based on the recent data is denoted by D, then the updated prediction is some weighted combination

$$ZD + (1 - Z)M$$

The value Z is called the **credibility factor**. If Z = 1, then we say that **full credibility** has been obtained. If 0 < Z < 1, then **partial credibility** has been obtained.

In the classical credibility approach, the minimum size of data required for full credibility is called **standard for full credibility**.

Full credibility in the general setting

Here we assume an i.i.d. $\{X_1, \ldots, X_n\}$ of random variables with mean μ and variance σ^2. In this case, we say that full credibility is attained if

$$p \le P\left(\left|\frac{\bar{X} - E(\bar{X})}{E(\bar{X})}\right| < r\right)$$

But, $E(\bar{X}) = \mu$ and $s(\bar{X}) = \frac{\sigma}{\sqrt{n}}$. So we can write:

$$p \le P\left(\left|\frac{\bar{X} - E(\bar{X})}{E(\bar{X})}\right| < r\right) = P(|\bar{X} - E(\bar{X})| < r E(\bar{X})) = P\left(\left|\frac{\bar{X} - E(\bar{X})}{\mathrm{Std}(\bar{X})}\right| < \frac{r E(\bar{X})}{\mathrm{Std}(\bar{X})}\right)$$

$$= P(|N(0,1)| < \frac{\sqrt{n}\, r\mu}{\sigma})$$

$$\Rightarrow \quad z_{\frac{1+p}{2}} \le \frac{\sqrt{n}\, r\mu}{\sigma} \quad \Rightarrow \quad n \ge \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left(\frac{\sigma}{\mu}\right)^2$$

Note. The value $\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2$ is denote by n_0 in the Mahlar-and Dean Note.

Example. Suppose that the estimates for mean and variance of the severity are 1000 and 2,000,000 respectively. Find the standard of full credibility for $p = 0.99$ and $r = 0.05$.

Solution.

$$z_{\frac{1+p}{2}} = z_{0.995} = 2.5758$$

$$\text{standard} = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left(\frac{\sigma}{\mu}\right)^2 = \left(\frac{2.5758}{0.05}\right)^2 \frac{\sigma^2}{\mu^2} = \left(\frac{2.5758}{0.05}\right)^2 \frac{2{,}000{,}000}{(1000)^2} = 5308$$

Full credibility for claim frequency

Let $\{N_1, \ldots, N_n\}$ be a sequence of independent identically distributed sequence of numbers of claims where each $N_i \sim \text{Poisson}(\lambda)$. We want to find the standard for full credibility in terms of the expected total number of claims.

Solution:

For Poisson:

$\mu = \sigma^2 = \lambda$

Putting these into general inequality for the standard of full credibility, we get:

$$n \geq n_0 \left(\frac{\sigma}{\mu}\right)^2 = n_0 \left(\frac{1}{\sqrt{\lambda}}\right)^2 = \frac{n_0}{\lambda} \quad \Rightarrow \quad n\lambda \geq n_0$$

$\Rightarrow \quad$ expected number of total claims $\geq n_0$

Note: Since λ is not known, then instead of the expected number of claims we use the observed number of claims. So, to have full credibility the observed number of claims must be at least $n_0 = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2$

Example 2.2.2 of the Mahler-and-Dean: For p =95% and for r= 5%, what is the number of claims required for Full Credibility for estimating the frequency?

Solution:

$$z_{\frac{1+p}{2}} = z_{\frac{1+0.95}{2}} = z_{0.975} = 1.96 \quad \Rightarrow$$

$$\text{standard for full credibility} = n_0 = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 = \left(\frac{1.96}{0.05}\right)^2 = 1537$$

Note: This analysis for finding the minimum number of observations to give full credibility to the observed total number of claims only if the following are met:

i) We are estimating frequency;

ii) Frequency is given by a Poisson process;

Example. An insurance company wants to assign full credibility to 800 claims or more. What is the required coverage probability for the number of claims to be within 8% of the true value. Assume that the claims frequency is Poisson and normal approximation applies (i.e. λ is large).

Solution.

$$800 = \left(\frac{z_{\frac{1+p}{2}}}{0.08}\right)^2 \quad \Rightarrow \quad z_{\frac{1+p}{2}} = 2.2627 \quad \Rightarrow \quad p = 97.63\%$$

Standard for the general case of Frequency (not just Poisson)

$$\text{standard for full credibility} = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \frac{\sigma_f^2}{\mu_f}$$

Exposures vs. Claims: If the standard is in terms of the number of claims, one can easily translate it into the number of exposures by dividing by the approximate expected claim frequency; the reverse action might be need instead.

Example 2.2.3 of the Mahler-and-Dean: E represents the number of homogeneous exposures in an insurance portfolio. The claim frequency *rate* per exposure is a random variable with mean $= 0.025$ and variance $= 0.0025$. A full credibility standard is devised that requires the observed sample frequency rate per exposure to be within 5% of the expected population frequency rate per exposure 90% of the time. Determine the value of E needed to produce full credibility for the portfolio's experience.

Solution:

$$\text{the number of claims for full credibility} = \left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 \frac{\sigma_{\mathrm{f}}^2}{\mu_{\mathrm{f}}} = \left(\frac{1.645}{0.05}\right)^2 \left(\frac{0.0025}{0.025}\right) = 108.241 \quad \text{claims}$$

Now divide by the claim frequency rate per exposure:

$$\text{the number of exposures for full credibility} = \frac{108.241}{0.025} = 4330 \quad \text{exposures}$$

Example. Recent experience has given the mean accident rate to be 0.045 and the standard for full credibility of claims to be 1200. For a group with similar risk, what is the number of exposure units for full credibility?.

Solution. The standard for full credibility based on exposure unit:

$$\frac{1200}{0.045} = 26,667 \quad \text{exposure units}$$

Example on the Binomial case (Exercise 2.2.5 of the Mahler-and-Dean): Assume you are conducting a poll relating to a single question and that each respondent will answer either yes or no. You pick a random sample of respondents out of a very large population. Assume that the true percentage of yes responses in the total population is between 20% and 80%. How many respondents do you need, in order to require that there be a 95% chance that the results of the poll are within ±7% of the true answer?

Solution: Since we want to approximate the population proportion with the sample proportion, we have the case of $\frac{X_1+\cdots+X_n}{n}$ where each X_i is assumed to be $\mathrm{Bernoulli}(p)$. The minimum umber for full credibility is:

$$z_{\frac{1+p}{2}} = z_{\frac{1+0.95}{2}} = z_{0.975} = 1.96 \qquad \Rightarrow$$

the number of claims for full credibility $= \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left(\frac{\sigma_{\text{Bernoulli}}}{\mu_{\text{Bernoulli}}}\right)^2 = \left(\frac{1.96}{0.07}\right)^2 \left(\frac{p(1-p)}{p^2}\right) =$
$784\left(\frac{1}{p}-1\right)$

Of course by ing a very large number our goal is achieved, and as p becomes smaller a larger value of n is needed. But since we don't know the true value of p, we must consider the worst scenario which is $p = 0.2$ in which case we get $n = 748(5-1) = 3136$. With this number of observations we will be 95% confident that for all $0.2 \leq p \leq 0.8$ the sample proportion is within ±7% of the population.

Full credibility for aggregate loss under the assumption of Poisson

Here we have $\mathrm{S} = \mathrm{X}_1 + \cdots + \mathrm{X}_\mathrm{N}$, where the $\mathrm{X_i}$'s have common mean μ_X and common variance σ_X^2. If N is Poisson, then

$$\begin{cases} \mu_\mathrm{S} = \mu_\mathrm{N}\mu_\mathrm{X} = \lambda\mu_\mathrm{X} \\ \sigma_\mathrm{S}^2 = \lambda(\mu_\mathrm{X}^2 + \sigma_\mathrm{X}^2) \end{cases}$$

If we apply the formula for the general case, we should have the following standard for full credibility:

$$\mathrm{n} \geq \mathrm{n}_0 \left(\frac{\sigma_\mathrm{S}}{\mu_\mathrm{S}}\right)^2 = \mathrm{n}_0 \frac{\sigma_\mathrm{S}^2}{\mu_\mathrm{S}^2} = \mathrm{n}_0 \frac{\lambda(\mu_\mathrm{X}^2 + \sigma_\mathrm{X}^2)}{(\lambda\mu_\mathrm{X})^2} \quad \Rightarrow \quad \mathrm{n}\lambda \geq \mathrm{n}_0 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\} = \left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\}$$

So, the expected number of observations must be at least equal to

$$\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\}$$

In practice, if n is the observed number of claims under the assumption of Poisson, then for full credibility we check for

$$n \geq \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\} = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{\frac{\mu^2 + \sigma^2}{\mu^2}\right\} = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{\frac{E(X^2)}{E(X)^2}\right\}$$

Note. This expression on the right-hand side is:

$$\left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\} = \left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 + \left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 \left(\frac{\sigma}{\mu}\right)^2$$

So for Poisson claim distribution we have :

standard for full credibility of aggregate loss =

standard for full credibility of claim frequency + standard for full credibility of claim severity

Example (exercise 17.7 of the textbook $*$). The average claim size for a group of insureds is 1,500 with a standard deviation of 7,500. Assume that claim counts have the Poisson distribution. Determine the expected number of claims so that the total loss will be within 6% of the expected total loss with probability 0.90.

Solution.

$z_{\frac{1+p}{2}} = z_{0.95} = 1.645$

$$\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\} = \left(\frac{1.645}{0.06}\right)^2 \left\{1 + \left(\frac{7500}{1500}\right)^2\right\} = 19543.51 \quad \Rightarrow \quad 19544 \text{ claims}$$

Full credibility for aggregate loss in the general case (not just Poisson)

$$\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \left\{\frac{\sigma_f^2}{\mu_f} + \frac{\sigma_X^2}{\mu_X^2}\right\} = n_0 \frac{\sigma_f^2 \mu_X^2 + \sigma_X^2 \mu_f}{\mu_f \mu_X^2} = n_0 \frac{\mathrm{Var}(S)}{\mu_f \mu_X^2} = n_0 \frac{\sigma_S^2}{\mu_S^2}\, \mu_f$$

= standard for full credibility of claim frequency + standard for full credibility of claim severity

Note. Here the exercise 2.5.6 of page 27 of Mahler-and-Dean was solved.

Example (SOA exam). You are given:

(i) The number of claims follows a negative binomial distribution with parameters r and $\beta = 3$.

(ii) Claim severity has the following distribution:

Claim Size	Probability
1	0.4
10	0.4
100	0.2

(iii) The number of claims is independent of the severity of claims.

Determine the expected number of claims needed for aggregate losses to be within 10% of expected aggregate losses with 95% probability.

A) Less than 1200

B) At least 1200, but less than 1600

C) At least 1600, but less than 2000

D) At least 2000, but less than 2400

E) At least 2400

Solution. Let S be the compound distribution with severity distribution X and frequency distribution N.

$E(N) = r\beta = 3r$

$Var(N) = r\beta(1+\beta) = 12r$

$E(X) = (1)(0.4) + (10)(0.4) + (100)(0.2) = 24.4$

$E(X^2) = (1)^2(0.4) + (10)^2(0.4) + (100)^2(0.2) = 2040.4$

$Var(X) = E(X^2) - E(X)^2 = 2040.4 - (24.4)^2 = 1445.04$

$E(S) = E(N)E(X) = (3r)(24.4) = 73.2r$

$Var(S) = E(N)Var(X) + Var(N)E(X)^2 = (3r)(1445.04) + (12r)(24.4)^2 = 11479.44r$

$z_{\frac{1+p}{2}} = z_{0.975} = 1.96$

standard for full credibility $= \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \cdot \frac{Var(S)}{E(S)^2} E(N) = \left(\frac{1.96}{0.1}\right)^2 \cdot \frac{11479.44}{(73.2r)^2}(3r) = 2469.1 \approx 2470$

Example (SOA exam - Fall 2005). You are given:

(i) The number of claims follows a Poisson distribution.

(ii) Claim sizes follow a gamma distribution with parameters α (unknown) and $\theta = 10,000$.

(iii) The number of claims and claim sizes are independent.

(iv) The full credibility standard has been selected so that actual aggregate losses will be within 10% of expected aggregate losses 95% of the time.

Using limited fluctuation (classical) credibility, determine the expected number of claims required for full credibility.

A) Less than 400

B) At least 400, but less than 450

C) At least 450, but less than 500

D) At least 500

E) The expected number of claims required for full credibility cannot be determined from the information given.

Solution. Let S be the compound distribution with Poisson parameter λ and severity distribution X. Then the expected number of claims required for full credibility is:

$$z_{\frac{1+p}{2}} = z_{0.975} = 1.96$$

$$\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \cdot \frac{\mathrm{Var}(S)\,E(N)}{E(S)^2} = \left(\frac{1.96}{0.1}\right)^2 \cdot \frac{(\lambda\,E(X^2))\,\lambda}{\lambda^2\,E(X)^2} = \frac{384.16(\alpha+1)}{\alpha}$$

This number cannot be determined as α is unknown.

Full credibility for pure premium

Pure premium $\mathrm{P} = \frac{\mathrm{S}}{\mathrm{E}}$, where the number of exposure units, E, is a constant, is the premium charged to cover losses before taking into consideration the profits and expenses. Since P and S differ by a constant multiple only, we have $\frac{\mu_\mathrm{P}}{\sigma_\mathrm{P}} = \frac{\mu_\mathrm{S}}{\sigma_\mathrm{S}}$, therefore

$$n \geq n_0 \left(\frac{\sigma_S}{\mu_S}\right)^2 \quad \Leftrightarrow \quad n \geq n_0 \left(\frac{\sigma_P}{\mu_P}\right)^2$$

so the standard for full credibility of the pure premium would be the same as that of the aggregate loss.

Example (exercise 17.13 of the textbook $*$). The number of claims has the Poisson distribution. The number of claims and the claim severity are independent. Individual claim amounts can be for 1, 2, or 10 with probabilities 0.5, 0.3, and 0.2, respectively. Determine the expected number of claims needed so that the total cost of claims is within 10% of the expected cost with 90% probability.

Solution.

$\mathrm{E(X)} = (0.5)(1) + (0.3)(2) + (0.2)(10) = 3.1$

$\mathrm{E(X^2)} = (0.5)(1)^2 + (0.3)(2)^2 + (0.2)(10)^2 = 21.7$

$\mathrm{Var(X)} = \mathrm{E(X^2)} - \mathrm{E(X)}^2 = 21.7 - (3.1)^2 = 12.09$

$\mathrm{z}_{\frac{1+\mathrm{p}}{2}} = \mathrm{z}_{0.95} = 1.645$

$$\left(\frac{\mathrm{z}_{\frac{1+\mathrm{p}}{2}}}{\mathrm{r}}\right)^2 \left\{1 + \left(\frac{\sigma}{\mu}\right)^2\right\} = \left(\frac{1.645}{0.01}\right)^2 \left\{1 + \left(\frac{\sqrt{12.09}}{3.1}\right)^2\right\} = 611.04 \quad \Rightarrow \quad 612 \text{ claims}$$

Example (SOA exam - Fall 2006). A company has determined that the limited fluctuation full credibility standard is 2000 claims if:

(i) The total number of claims is to be within 3% of the true value with probability p.

(ii) The number of claims follows a Poisson distribution.

The standard is changed so that the total cost of claims is to be within 5% of the true value with probability p, where claim severity has probability density function:

$$f(x) = \frac{1}{10,000}, \qquad 0 \le x \le 10,000$$

Using limited fluctuation credibility, determine the expected number of claims necessary to obtain full credibility under the new standard.

(A) 720

(B) 960

(C) 2160

(D) 2667

(E) 2880

Solution. **Note** that the value of p is not given. And that's why the information in the first part of the question is given to you to use it for the second part.

Let the severity distribution be X and the frequency distribution be N. The standard for fulll credibility in case of Poisson is:

$$2000 = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 = \left(\frac{z_{\frac{1+p}{2}}}{0.03}\right)^2 \quad \Rightarrow \quad \left(z_{\frac{1+p}{2}}\right)^2 = 1.8$$

for uniform distribution: $\mathrm{E(X)} = \dfrac{a+b}{2} = 5000$

$$\mathrm{Var(X)} = \frac{(b-a)^2}{12} = \frac{10000}{12}$$

$$\mathrm{E(X^2)} = \mathrm{Var(X)} + \mathrm{E(X)}^2 = \frac{10,000}{12} + 25,000,000 = 25,000,833$$

Now, for $r = 0.05$ we get:

$$2000 = \left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 \frac{E(X^2)}{E(X)^2} = \frac{1.8}{(0.05)^2}\frac{25,000,833}{(5,000)^2} = 960$$

Note. Here the example 2.5.1 of page 24 of Mahler-and-Dean was solved.

Note. Here the exercise 2.5.1 of page 26 of Mahler-and-Dean was solved.

Note. Here the exercise 2.5.3 of page 26 of Mahler-and-Dean was solved.

Note. Here the exercise 2.5.5 of page 27 of Mahler-and-Dean was solved.

Note. Here the exercise 2.5.8 of page 28 of Mahler-and-Dean was solved.

Partial Credibility

Assume that W is any of the loss measures claim frequency, claim severity, or aggregate loss. When the risk group is not large enough, the full credibility may not be achieved in which case a combination $ZW+(1-Z)M$ is taken, where $0<Z<1$. The number Z is determined such that the event $|ZW-ZE(W)|\leq rE(W)$ occur with probability p. For example, for the case of average claims frequency $N=\frac{N_1+\cdots+N_n}{n}$, where each N_i follows Poisson(λ), we want to have

$$p=P(|ZN-ZE(N)|<rE(N))=P\left(\left|\frac{N-E(N)}{s(N)}\right|<\frac{rE(N)}{Zs(N)}\right)=P(|N(0,1)|<\frac{r\mu}{Z\sigma})$$

$$\Rightarrow \quad z_{\frac{1+p}{2}}=\frac{r\mu_N}{Z\sigma_N}=\frac{r\lambda}{Z\frac{\sqrt{\lambda}}{\sqrt{n}}}=\frac{r\sqrt{n\lambda}}{Z} \quad \Rightarrow \quad Z=\left(\frac{r}{z_{\frac{1+p}{2}}}\right)\sqrt{n\lambda}=\sqrt{\frac{n\lambda}{\text{standard for frequency}}}=\sqrt{\frac{\text{expected number of claims}}{\text{standard for frequency}}}$$

Note. For all cases of **Frequency**, **severity**, **aggregate loss**, and **pure premium**, the credibility weight Z under the Classical Credibility Theory is calculate through the formula

$$\sqrt{\frac{n}{\text{standard}}}$$

where n is the expected number of claims or the observed number of claims, whichever available (use the expected number of claims if it is available). When calculating Z for the **aggregate loss** and **pure premium** first calculate the standards for both the frequency and severity and then add them up to get the standard for the aggregate loss or pure premium. See the second example below.

Note. There is a different justification (more comprehensive than the one presented above) for this formula on pages 30, 31, and 32 of the Mahler-and-Dean lecture note.

Example (exercise 17.8 of the textbook $*$). A group of insureds had 6,000 claims and a total loss of 15,600,000. The prior estimate of the total loss was 16,500,000. Determine the limited fluctuation credibility estimate of the total loss for the group. Use the standard for full credibility determined in the exercise 17.7 of the textbook solved above.

Solution.

$$\sqrt{\frac{n\lambda}{\text{standard}}} = \sqrt{\frac{6000}{19543.51}} = 0.55408$$

$$ZW + (1-Z)M = (0.55408)(15,600,000) + (1-0.55408)(16,500,000) = 16,001,328$$

Example. A portfolio of policies has 896 claims in the current period with mean loss of 45 and variance being 5067. Full credibility is based on a coverage probability of 98% for a range of within 10% of the true mean. The mean frequency of claims is 0.09 per policy and the portfolio has 18600 policies. Calculate Z for the claim frequency, claim severity, and aggregate loss.

Solution.

Part 1.

expected claim frequency for the portfolio $= (18600)(0.09) = 1674$

$$z_{\frac{1+p}{2}} = z_{0.99} = 2.3263$$

Full credibility standard for the claim frequency:

$$\left(\frac{z_{\frac{1+p}{2}}}{r}\right)^2 = \left(\frac{2.3263}{0.1}\right)^2 = 541.17 < 1674 \quad \Rightarrow \quad \text{full credibility for claim frequency} \quad \checkmark$$

Part 2.

Coefficient of variation for claim severity:

$$C_X = \frac{\sqrt{5067}}{45} = 1.5818$$

Full credibility standard for claim severity:

standard for claim frequency times $C_X^2 = (541.17)(1.5818)^2 = 1354.13 > 896$

$\Rightarrow$ partial credibility for claim severity ✓

Now we calculate the partial credibility factor for claim severity:

$$Z = \sqrt{\frac{896}{1354.13}} = 0.8134$$

Part 3.

The full credibility standard for aggregate loss:

sum of two standards found above $= 541.17 + 1354.13 = 1895.30 > 1674 \quad \Rightarrow$
partial credibility for aggregate loss

Partial credibility factor for aggregate claim:

$$Z = \sqrt{\frac{1674}{1895.30}} = 0.9398$$

Example (Spring 1996). You are given the following:

i) The number of claims follows a Poisson distribution.

ii) Claim sizes follow a gamma distribution with parameters $\alpha = 1$ and θ unknown.

iii) The number of claims and claim sizes are independent.

The full credibility standard has been selected so that the actual number of claims will be within 5% of the expected number of claims 90% of the time. Using the methods of classical credibility as described in Mahler and Dean, determine the credibility to be given to the experience if 500 claims are expected.

Solution.

$$z_{\frac{p+1}{2}} = z_{0.95} = 1.645$$

$$\text{standard} = \left(\frac{z_{\frac{p+1}{2}}}{r}\right)^2 = \left(\frac{1.645}{0.05}\right)^2 = 1082$$

$$Z = \sqrt{\frac{n}{\text{standard}}} = \sqrt{\frac{500}{1082}} = 68\%$$

Note. Here the exercise 2.6.3 of page 32 of Mahler-and-Dean was solved.

Note. Here the exercise 2.6.4 of page 33 of Mahler-and-Dean was solved.

Note. Here the exercise 2.6.5 of page 33 of Mahler-and-Dean was solved.

Note. Here the exercise 2.6.8 of page 34 of Mahler-and-Dean was solved.

Note. Here the exercise 2.6.9 of page 34 of Mahler-and-Dean was solved.

Note. Here the questions 2, 39, 65, 148, and 273 of the "SOA Sample Questions" were solved in class.

Talk about the Central Limit Theorem and the classical credibility for the cases $\bar{X}$ and the aggregate loss S