P.1

1- Section 1.5 Exercise 77: Draw the curve $y=|3-|x+2||-1$ and indicate whether it defines y as a function of x.

Sol. As a general rule, ~~once~~ y stands alone on one side of the equation, it defines a function. So, the above equality defines a function $y=f(x)$ where $f(x)=|3-|x+2||-1$.

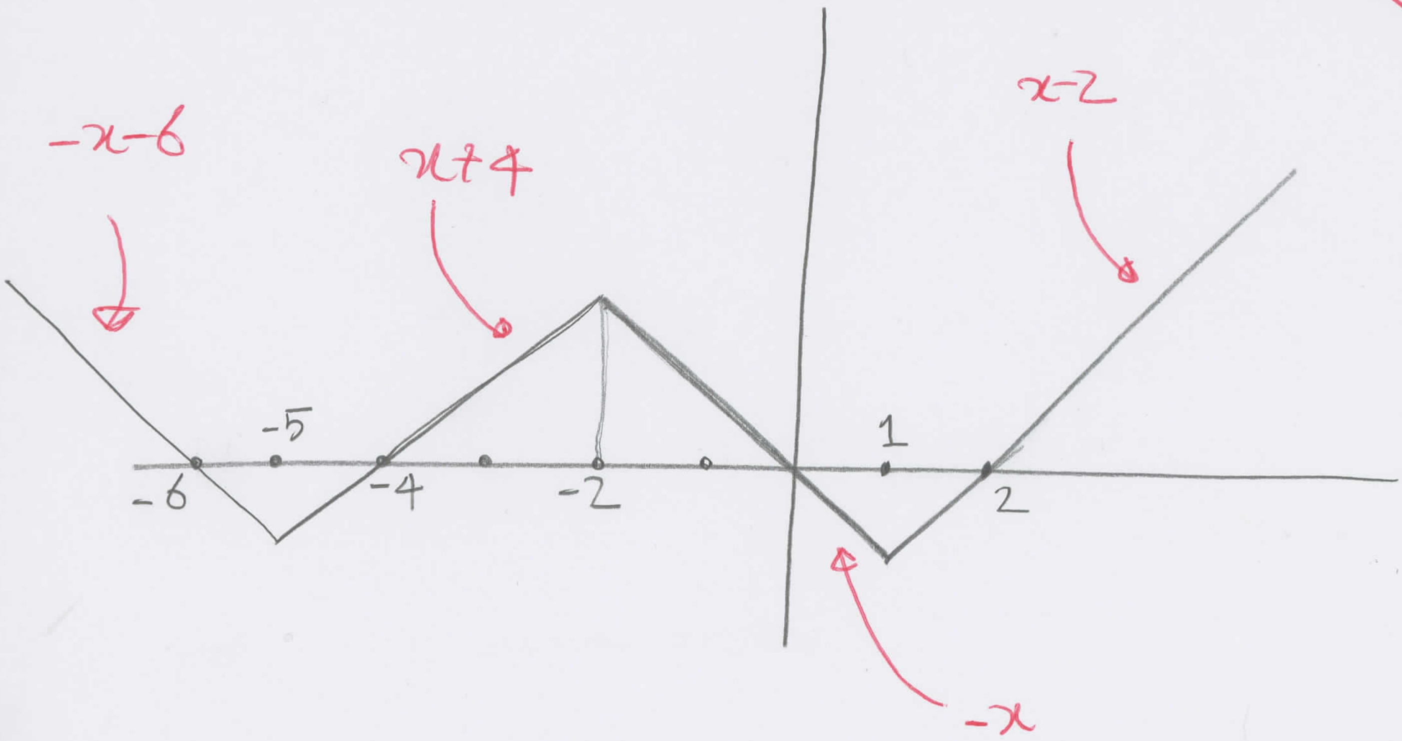
$$y=\begin{cases} |3-(x+2)|-1 & x+2\geq 0 \\ |3+(x+2)|-1 & x+2<0 \end{cases}$$

$$\Rightarrow y=\begin{cases} |1-x|-1 & x\geq -2 \\ |5+x|-1 & x<-2 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} \begin{cases} (1-x)-1 & x \geq -2 \ \& \ 1-x \geq 0 \\ -(1-x)-1 & x \geq -2 \ \& \ 1-x < 0 \end{cases} \\ \begin{cases} (5+x)-1 & x < -2 \ \& \ 5+x \geq 0 \\ -(5+x)-1 & x < -2 \ \& \ 5+x < 0 \end{cases} \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} -x & x \geq -2 \ \& \ 1 \geq x \\ x-2 & x \geq -2 \ \& \ 1 < x \\ x+4 & x < -2 \ \& \ x \geq -5 \\ -x-6 & x < -2 \ \& \ x < -5 \end{cases}$$

$$\Rightarrow \quad y = \begin{cases} -x & -2 \leq x \leq 1 \\ x-2 & 1 < x \\ x+4 & -5 \leq x < -2 \\ -x-6 & x < -5 \end{cases}$$



Section 1.5 exercise 78

From $|x| = 1+|y|$ we have $|x| \geq 1$, so equivalently $\pm x \geq 1$, equivalently $\left(\begin{array}{l} x\geq 1 \text{ or} \\ -x \geq 1 \end{array}\right)$, equivalently ($x \geq 1$ or $x \leq -1$). Therefore the graph of this curve is allowed on the domain $D = \{x : x \geq 1 \text{ or } x \leq -1\}$ only. Furthermore:

Case: $x \geq 1$ $\xRightarrow{|x|-|y|=1}$ $x-|y|=1 \Longrightarrow$

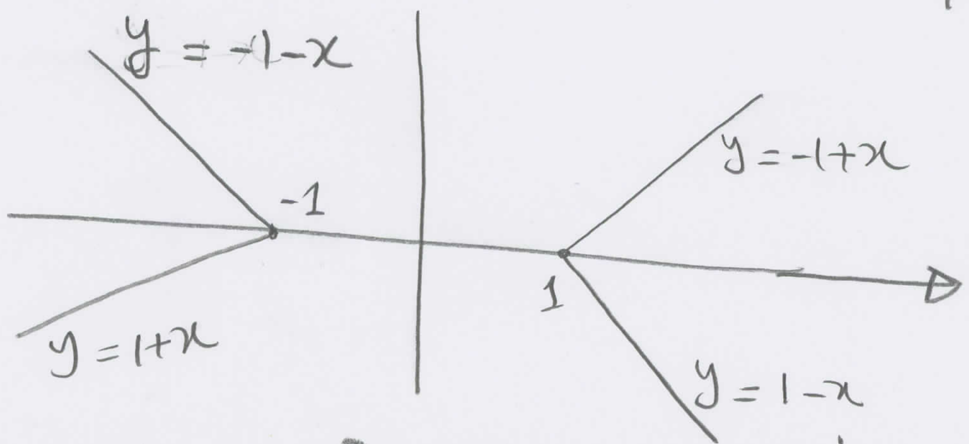
$x-(\pm y)=1 \Longrightarrow x \mp y = 1 \Longrightarrow \mp y = 1-x$

$\Longrightarrow y = \mp(1-x) \Longrightarrow y = \begin{cases} 1-x & x \geq 1 \\ -1+x & x \geq 1 \end{cases}$

Case: $x \leq -1$ $\xRightarrow{|x|-|y|=1}$ $-x-|y|=1 \Longrightarrow$

$-x-(\pm y)=1 \Longrightarrow -x \mp y = 1 \Longrightarrow$

$\mp y = 1+x \Longrightarrow y = \mp(1+x) \Longrightarrow y = \begin{cases} 1+x & x \leq -1 \\ -1-x & x \leq -1 \end{cases}$



this is not a function as the graph shows

Section 2.3 Exercise 15

$$\lim_{x\to\infty} (x^2 - x^3) = \lim_{x\to\infty} x^3\left(\frac{1}{x} - 1\right)$$

$$= \lim_{x\to\infty} x^3\left(\frac{1}{x} - 1\right) = (\infty)(0-1) = -\infty$$

Section 2.3 Exercise 16

$$\lim_{x\to\infty} \left(x + \frac{1}{x}\right) = \infty + 0 = \infty$$

Section 2.3 Exercise 20

$$\lim_{x\to\infty} \frac{3x}{\sqrt[3]{2+4x^3}} = \frac{3}{2}$$

$$= \lim_{x\to\infty} \frac{3x}{\sqrt[3]{x^3\left(\frac{2}{x^3}+4\right)}} = \lim_{x\to\infty} \frac{3x}{x\sqrt{\frac{2}{x^3}+4}} = \lim \frac{3}{\sqrt{\frac{2}{x^3}+4}}$$

$$= \frac{3}{2}$$

34. $\lim_{x\to-\infty} \frac{\sqrt{4x^2+7}}{2x+3} = \lim_{x\to-\infty} \frac{\sqrt{x^2(4+\frac{7}{x^2})}}{2x+3}$

$= \lim_{x\to-\infty} \frac{|x|\sqrt{4+\frac{7}{x^2}}}{2x+3} = \lim_{x\to-\infty} \frac{-\cancel{x}\sqrt{4+\cancel{\frac{7}{x^2}}}}{\cancel{x}(2+\cancel{\frac{3}{x}})} = \frac{\sqrt{4}}{2} = 1$

$|x| = x \quad x\to+\infty \qquad |x| = -x \quad x\to-\infty$

35. $\lim_{x\to\infty} \sqrt{x^2+4} - \sqrt{x^2-1} = \lim_{x\to\infty} \frac{(\sqrt{x^2+4}-\sqrt{x^2-1})(\sqrt{x^2+4}+\sqrt{x^2-1})}{\sqrt{x^2+4}+\sqrt{x^2-1}}$

$= \lim_{x\to\infty} \frac{(x^2+4)-(x^2-1)}{\sqrt{x^2+4}+\sqrt{x^2-1}} = \lim \frac{5}{|x|\sqrt{1+\frac{4}{x^2}} + |x|\sqrt{1-\frac{1}{x^2}}}$

$= \lim_{x\to\infty} \frac{5}{2x} = 0$

36. $\lim_{x\to\infty} \sqrt[3]{1+x} - \sqrt[3]{x} = \lim_{x\to\infty} \frac{(\sqrt[3]{1+x}-\sqrt[3]{x})\left[(\sqrt[3]{1+x})^2 + \sqrt[3]{1+x}\sqrt[3]{x} + (\sqrt[3]{x})^2\right]}{\left[(\sqrt[3]{x+1})^2 + \sqrt[3]{x}\sqrt[3]{x+1} + (\sqrt[3]{x})^2\right]}$

$= \lim_{x\to\infty} \frac{(1+\cancel{x}) - \cancel{x}}{(\sqrt[3]{x+1})^2 + \sqrt[3]{x}\sqrt[3]{x+1} + \sqrt[3]{x}} = \frac{1}{\infty} = 0$

37. $\lim_{x\to\infty} (\sqrt{x^2+x} - x) = \lim_{x\to\infty} \frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$

$= \lim_{x\to\infty} \frac{\cancel{x^2}+x-\cancel{x^2}}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\cancel{|x|}\sqrt{1+\frac{1}{x}} + x}$

$= \lim_{x\to\infty} \frac{\cancel{x}}{2\cancel{x}} = \frac{1}{2}$

3. Calculate $\lim_{n\to\infty} \sqrt{(n+1)(n+2)} - n$

$$\lim_{x\to\infty} \frac{\left[\sqrt{(x+1)(x+2)} - x\right] \times \left[\sqrt{(x+1)(x+2)} + x\right]}{\sqrt{(x+1)(x+2)} + x}$$

$$= \lim_{x\to\infty} \frac{(x+1)(x+2) - x^2}{\sqrt{x^2+3x+2} + x} = \lim_{x\to\infty} \frac{\cancel{x^2} + 3x + 2 - \cancel{x^2}}{|x|\sqrt{1+\frac{3}{x}+\frac{2}{x^2}} + x}$$

$$= \lim_{x\to\infty} \frac{3x+2}{2x} = \frac{3}{2}$$

4. Calculate $\lim_{x\to\infty} x^{\frac{3}{2}}\left(\sqrt{x^3+1} - \sqrt{x^3-1}\right)$

$$= \lim_{x\to\infty} \frac{x^{3/2}\left(\sqrt{x^3+1} - \sqrt{x^3-1}\right)\left(\sqrt{x^3+1} + \sqrt{x^3-1}\right)}{\sqrt{x^3+1} + \sqrt{x^3-1}}$$

$$= \lim_{x\to\infty} \frac{x^{3/2}\left[(\cancel{x^3}+1) - (\cancel{x^3}-1)\right]}{\sqrt{x^3+1} + \sqrt{x^3 1}} = \lim_{x\to\infty} \frac{2x^{3/2}}{2\sqrt{x^3}} = 1$$

5. Use limits to determine the values of a anb b so that the function defined by

$$f(x) = \begin{cases} ax & x < -1 \\ 2 & x = -1 \\ x^2 + b & x > -1 \end{cases}$$

is contnous for all real numbers.

Continuous at $x = a$ when

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1} x^2 + b = 1 + b$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1} ax = a(-1) = -a$$

$$f(-1) = 2 \Rightarrow 1 + b = -a = 2 \qquad 1 + b = 2 \Rightarrow \boxed{b = 1}$$

$$-a = 2 \qquad \boxed{a = -2}$$

6. Use the Synthetic division to devide the polynomial $p(x) = 4x^5 + 3x^3 - 2x^2 + 1$ by the linear factor $x+3$

$x + 3 = 0 \Rightarrow x = -3$

	4	0	3	-2	0	1
-3		-12	36	-117	357	-1071
product	4	-12	39	-119	357	1070

add

$\hookrightarrow -3 \times 4 = -12 + 0 = -12$

remainder

devider

$$\frac{p(x)}{x+3} \quad 4x^4 - 12x^3 + 39x^2 - 119x + 357$$

quotient

$$\vdots$$

1070

or we could write as

$$4x^5 + 3x^3 - 2x^2 + 1 = (x+3)(4x^4 - 12x^3 + 39x^2 - 119x + 357) + 1070$$