Solutions to some of Lab 9 questions

<u>Solution to section 4.3 exercise 18</u>.

$f'(x) = \frac{\{(x-1)^3\}'\{(x+1)^4\}-\{(x-1)^3\}\{(x+1)^4\}'}{(x+1)^8}$

$= \frac{\{3(x-1)^2\}\{(x+1)^4\}-\{(x-1)^3\}\{4(x+1)^3\}}{(x+1)^8}$

$= \frac{(x-1)^2(x+1)^3\{3(x+1)-4(x-1)\}}{(x+1)^8}$

$= \frac{(x-1)^2(x+1)^3\{-x+7\}}{(x+1)^8}$

$= \frac{(x-1)^2\{-x+7\}}{(x+1)^5}$

$f'(x) = 0 \quad \Rightarrow \quad x = 1\ ,\ 7$

This derivative does not exist at $x = -1$ but this point is not in the domain , therefore the only critical points are $x = 1$ and $x = 7$.

Since the term $(x-1)^2$ does not affect the sign of f' , we do not put the point $x = -1$ in the table for the analysis of the sign of f'. On the other hand, note that the term $(x+1)^5$ affects the sign of f' therefore we should put the point $x = -1$ in the table although this point is not in the domain.

		-1		7	
$x+1$	$-$	$\bullet$	$+$		$+$
$-x+7$	$+$		$+$	$\bullet$	$-$
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$
				relative max	

Note: Be careful that the point $x = -1$ is not considered as a relative minimum point because it is not in the domain.

Solution to section 4.3 exercise 26.

$$f'(x) = \frac{\{x^2\}'\{x^2-4\}-\{x^2\}\{x^2-4\}'}{(x^2-4)^2}$$
$$= \frac{\{2x\}\{x^2-4\}-\{x^2\}\{2x\}}{(x^2-4)^2}$$
$$= \frac{-8x}{(x^2-4)^2}$$

$$f'(x) = 0 \quad \Rightarrow \quad x = 0$$

$$f'(x) = 0 \quad \Rightarrow \quad x = 0$$

This derivative does not exist at $x = \pm 2$ but these points are not in the domain , therefore the only critical points is $x = 0$.

		0	
$-8x$	$+$	$\bullet$	$-$
f'	$+$	$\bullet$	$-$
f	$\nearrow$		$\searrow$
		relative max	

Solution to section 4.4 exercise 2.

$f'(x) = 12x^3 + 12x^2 - 24$

$f''(x) = 36x^2 + 24x = 12x(3x + 2)$ $f''(x) = 0 \quad \Rightarrow \quad x = 0\ ,\ -\frac{2}{3}$

There are no points where f'' does not exist , so there is no other candidates.

		$-\frac{2}{3}$		0	
$3x+2$	$-$	$\bullet$	$+$		$+$
x	$-$		$-$	$\bullet$	$+$
f''	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$
		inflection point		inflection point	

Solution to section 4.4 exercise 4.

$f'(x) = \cdots\cdots = \frac{-10x}{(x^2-1)^2}$

$f''(x) = \cdots\cdots = \frac{10(3x^2+1)}{(x^2-1)^3}$

Note that f'' never becomes zero. However it is not defined at the points $x = \pm 1$, so a non-careful student may think that these points are candidates for being inflection points , but the point is that these points are not in the domain of f. So there are no inflection points for this example. It only remains to describe the concavity of f. For this, the sign of the term $x^2 - 1 = (x-1)(x+1)$ must be determined in order to find the sign of f'' on different intervals.

		-1		1	
$x+1$	$-$	•	$+$		$+$
$x-1$	$-$		$-$	•	$+$
f''	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$

Solution to section 4.4 exercise 10.

$f'(x) = \{x^2\}'\{\ln x\} + \{x^2\}\{\ln x\}'$

$= \{2x\}\{\ln x\} + \{x^2\}\{\frac{1}{x}\}$

$= \{2x\}\{\ln x\} + \{x\}$

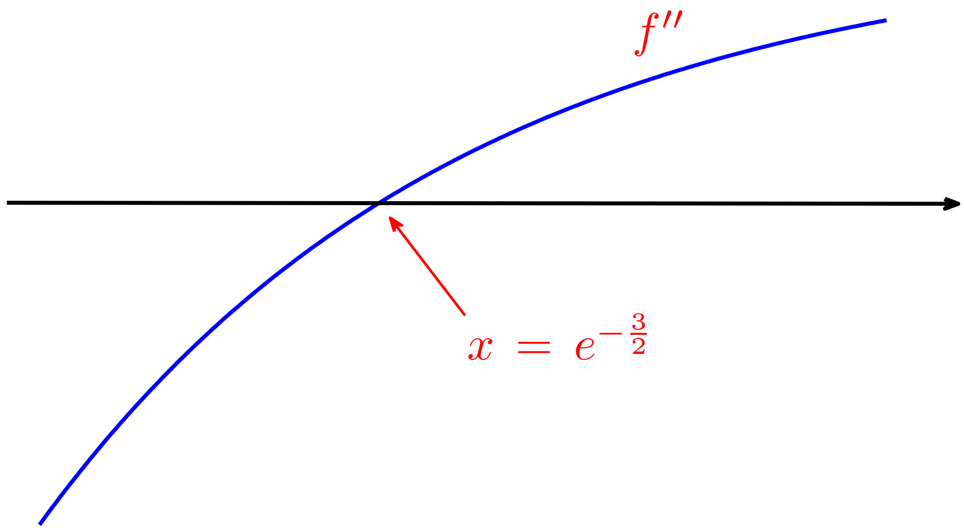
$f''(x) = \{2x\}'\{\ln x\} + \{2x\}\{\ln x\}' + \{x\}'$

$= \{2\}\{\ln x\} + \{2x\}\{\frac{1}{x}\} + \{1\}$

$= 2\ln x + 3$

$f''(x) = 0 \quad \Rightarrow \quad 2\ln x + 3 = 0 \quad \Rightarrow \quad \ln x = -\frac{3}{2} \quad \Rightarrow \quad x = e^{-\frac{3}{2}}$

Since the function $f''(x) = 2\ln x + 3$ is increasing (just note that its derivative is positive) on its domain $(0\,,\,\infty)$, and since it becomes zero at $x = e^{-\frac{3}{2}}$, its graph looks something like this when passes through the point $x = e^{-\frac{3}{2}}$:



We have the following table to analyze the sign of f''

		$e^{-\frac{3}{2}}$	
f''	$-$	$\bullet$	$+$
f	$\cap$		$\cup$

inflection point

So , the function f is concave down on $(0\,,\,e^{-\frac{3}{2}}]$ and is concave up on $[e^{-\frac{3}{2}}\,,\,\infty)$. The point $x = e^{-\frac{3}{2}}$ is the only inflection point.

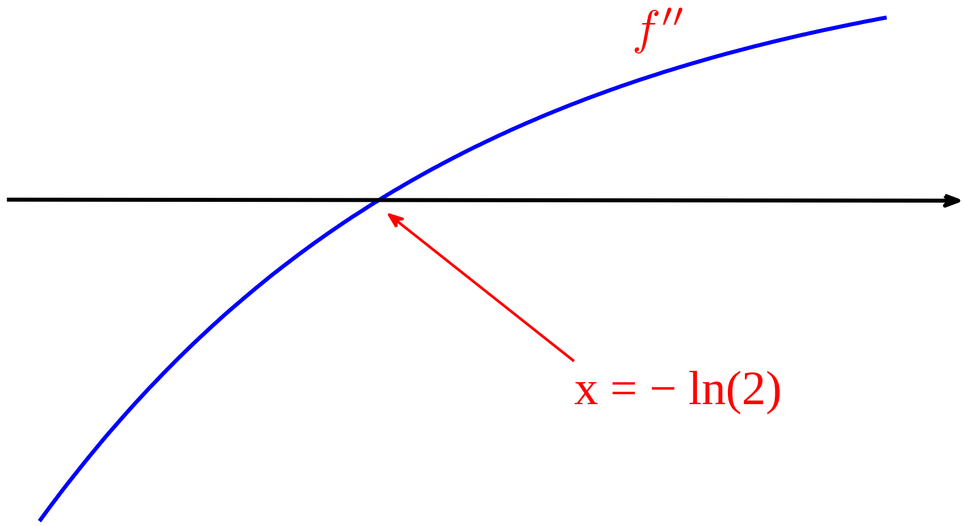
Solution to section 4.4 exercise 14.

$f'(x) = 2x + e^{-x}$

$f''(x) = 2 - e^{-x}$

$f''(x) = 0 \quad \Rightarrow \quad e^{-x} = 2 \quad \Rightarrow \quad -x = \ln(2) \quad \Rightarrow \quad x = -\ln(2)$

Since the function e^{x} is increasing , the function e^{-x} is decreasing , so then the function $-e^{-x}$ and then the function $2 - e^{-x}$ is increasing (one could alternatively differentiate this function to see that it is increasing), i.e. the function $f''(x) = 2 - e^{-x}$ is increasing and is zero at $-\ln(2)$; therefore its graph looks something like this when passes through the point $x = -\ln(2)$:



We have the following table to analyze the sign of f''

		$-\ln(2)$	
f''	$-$	$\bullet$	$+$
f	$\cap$	|	$\cup$

inflection point

The function is concave up on $(-\infty\,,\,-\ln(2)]$ and is concave up on the interval $[-\ln(2)\,,\,+\infty)$. The point $x = -\ln(2)$ in the domain is the only point of inflection.