Derivatives of Polynomials and Exponential Functions
(sections 3.1)

Theorem (Derivative of a Constant Function):

$$\frac{d}{dx}(c) = 0$$

Proof : The proof is given in another file.

The Power Rule (The General Version): If b is any real number, then

$$\frac{d}{dx}\left(x^b\right) = b\,x^{b-1}$$

Important Example :

$$\frac{d}{dx}(x) = \frac{d}{dx}(x^1) = 1\,x^0 = (1)(1) = 1 \quad \Rightarrow \quad \boxed{\tfrac{d}{dx}(x) = 1}$$

In fact, this is a special case of the Power Rule for $b = 1$.

Important Note : When you want to differentiate a function that involves root functions, convert every root function to a power function. See the example below:

Example : Find the derivative of the following functions:

(a) $y = x^{10}$

(b) $y = \sqrt[3]{x^2}$

(c) $A(s) = \dfrac{1}{s^{9.7}}$

Solution:

Solution to part (a) :

$$y = x^{10} \qquad \Rightarrow \qquad \frac{dy}{dx} = 10\,x^{10-1} = 10\,x^9$$

Solution to part (b) :

$$y = x^{\frac{2}{3}} \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{2}{3}\,x^{\frac{2}{3}-1} = \frac{2}{3}\,x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

Solution to part (c) :

$$A(s) = s^{-9.7} \qquad \Rightarrow \qquad A'(s) = (-9.7)\,s^{-9.7-1} = (-9.7)\,s^{-10.7} = -\,\frac{9.7}{s^{10.7}}$$

Theorem (The Constant Multiple Rule): If c is a constant and f is a differentiable function , then

$$\frac{d}{dx}\Big(cf(x)\Big) = c\,\frac{d}{dx}\,f(x)$$

symbolically:

$$(c\,f)' = c\,f'$$

Proof : The proof is given in another file.

Note: You are required to know the proof of the theorem "The Constant Multiple Rule"

Theorem (The Sum Rule): If f and g are differentiable , then

$$\frac{d}{dx}\Big[f(x) + g(x)\Big] = \frac{d}{dx}\,f(x) + \frac{d}{dx}\,g(x)$$

Equivalently:

$$(f+g)'(x) = f'(x) + g'(x)$$

Symbolically:

$$(f+g)' = f' + g'$$

Proof : The proof is given in another file.

Note: You are required to know the proof of the theorem "The Sum Rule"

Theorem (The Difference Rule): If f and g are differentiable , then

$$\frac{d}{dx}\Big[f(x) - g(x)\Big] = \frac{d}{dx}\,f(x) - \frac{d}{dx}\,g(x)$$

Derivative of the Natural Exponential Function :

$$\boxed{\frac{d(e^x)}{dx} = e^x}$$

Example (Midterm - Summer 2016): Differentiate the function

$f(x) = x^{-\frac{4}{5}} + x^3 + 2x - e^{\pi}$; Do Not Simplify Your Answer.

Solution:

$$\begin{aligned} f'(x) &= (x^{-\frac{4}{5}})' + (x^3)' + 2(x)' - (e^{\pi})' \\ &= -\frac{4}{5}x^{-\frac{9}{5}} + 3x^2 + 2 - 0 \end{aligned}$$

Example (section 3.1 - exercise 37): Find the equation of the tangent line to the curve $y = x^4 + 2e^x$ at the point $(0\,,\,2)$

Solution:

$$
\begin{aligned}
y' &= (x^4)' + 2(e^x)' \\
&= 4x^3 + 2e^x
\end{aligned}
$$

Put $x = 0$ to find the slope at $x = 0$

(slope) $m = 0 + 2e^0 = 0 + 2 = 2$

Equation of the tangent line:

$y - y_0 = m(x - x_0) \quad \Rightarrow \quad y - 2 = 2(x - 0) \quad \Rightarrow \quad y = 2x + 2$