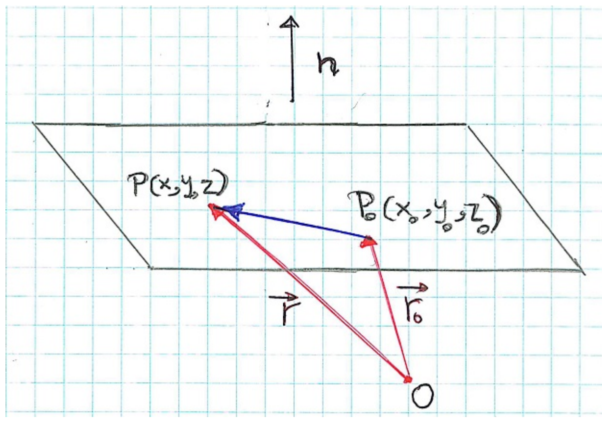
Equation of Plane

Here we show that by having a point $P_0(x_0, y_0, z_0)$ of a plane and a vector n orthogonal to the plane , called a **normal vector** , we can write the equation of the plane. In fact, if $P(x, y, z)$ is an arbitrary point of the plane , then the vector $\overrightarrow{P_0P}$ is on the plane , so it must be perpendicular to the plane, so $n \cdot \overrightarrow{P_0P} = 0$, so

$$\langle a\,,b\,,c\rangle \cdot \langle x - x_0\,, y - y_0\,, z - z_0\rangle = 0 \quad \Rightarrow \quad \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$



Example. Find the equation of the plane through $(4\,,0\,,-3)$ with normal vector $j + 2k$

Solution.

$$0(x - 4) + 1(y - 0) + 2(z + 3) = 0 \quad \Rightarrow \quad y + 2z + 6 = 0$$

Example. Give a candidate for the normal vector of the plane $2x - y + 3z = 12$

Solution. $\quad 2i - j + 3k \quad \checkmark$

Example. Find the equation of the plane through the point $(1\,,\frac{1}{2}\,,\frac{1}{3})$ and parallel to the plane $x + y + z = 0$

Solution. The normal vector of this plane can serve as the normal vector of the other one as the two plane are parallel.

$$1(x-1)+1(y-\frac{1}{2})+1(z-\frac{1}{3})=0 \quad \Rightarrow \quad x+y+z-\frac{11}{6}=0$$

Example. Find the equation of the plane through the origin and the points $(2\,,-4\,,6)$ and $(5\,,1\,,3)$

Solution. We are seeking the plane through the points:

$O(0\,,0\,,0) \qquad A(2\,,-4\,,6) \qquad B(5\,,1\,,3)$

Now:

$\overrightarrow{OA}=\langle 2\,,-4\,,6\rangle \qquad \overrightarrow{OB}=\langle 5\,,1\,,3\rangle$

$$(\textbf{normal vector}) \quad \overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix} i & j & k \\ 2 & -4 & 6 \\ 5 & 1 & 3 \end{vmatrix}=\langle -18\,,\ 24\,,\ 22\rangle$$

$$-18(x-0)+24(y-0)+22(z-0)=0 \quad \Rightarrow \quad -18x+24y+22z=0 \quad \Rightarrow \quad -9x+12y+11z=0$$