Rolle's Theorem and The Mean-Value Theorems
Section 3.14

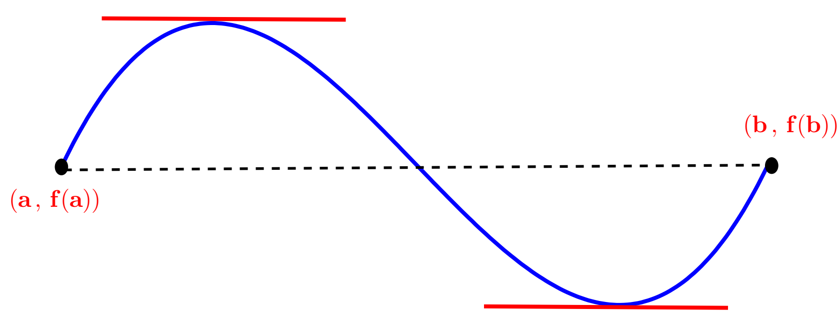
Rolle's Theorem. Suppose that function f satisfies the followings:

(a) f is continuous on the interval $[a, b]$

(b) f is differentiable at the interior points, i.e. it is differentiable at all points of the open interval (a, b)

(c) $f(a) = f(b)$

Then under these conditions , there exists some c such that $a < c < b$ and $f'(c) = 0$, i.e. somewhere over the interval (a, b) the tangent line is horizontal.



Note. This Rolle's Theorem is applied to prove the celebrated Mean-Value Theorem ; however we do not have to see how this is done:

Mean Value Theorem. Suppose that function f satisfies the followings:

(a) f is continuous on the interval $[a, b]$

(b) f is differentiable at the interior points, i.e. it is differentiable at all points of the open interval (a, b)

Then under these conditions , there exists some c such that $a < c < b$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

i.e. the tangent line at the point $(c.f(c))$ is parallel to the line joining the points $(a, f(a))$ and $(b, f(b))$