Synthetic Division

Example: One can easily check that the point $x = 1$ is a root of the polynomial

$$p(x) = 5x^6 - 4x^5 + 6x^4 - 18x^3 + 19x^2 - 9x + 1$$

So this polynomial is divisible by the linear factor $x - 1$. We now use the so called **synthetic division** to find the polynomial $q(x)$ satisfying $p(x) = (x - 1)q(x)$.

Solution:

	5	-4	6	-18	19	-9	1	
1		5	1	7	-11	8	-1	
	5	1	7	-11	8	-1	$\boxed{0}$	← remainder

So $q(x) = 5x^5 + x^4 + 7x^3 - 11x^2 + 8x - 1$

Example: Divide the polynomial $p(x) = 4x^5 - 3x^4 + 2x^2 + 1$ by the linear polynomial $x + 2$ (we do not know at this time whether the polynomial is divisible by $x + 2$).

	4	-3	0	2	0	1
-2		$\boxed{-8}$	22	-44	84	-168
	$\boxed{4}$	-11	22	-42	84	-167

$$(-2)(4) = -8$$

So the remainder is -167 and we have

$$4x^5 - 3x^4 + 2x^2 + 1 = (x + 2)(4x^4 - 11x^3 + 22x^2 - 42x + 84) - 167$$

Long Division

Example: Divide $3x^4 + x^2 - x + 1$ by $x^2 + 1$

Solution: Set $p_0(x) = 3x^4 + x^2 - x + 1$ and $q(x) = x^2 + 1$.

Step 1. Divide the highest degree term of $p_0(x)$ by that of $q(x)$. We need $\boxed{3x^2}$

multiply $q(x)$ by $3x^2$ $\quad\Rightarrow\quad$ $3x^4 + 3x^2$ $\quad\overset{\text{change sign}}{\Rightarrow}\quad$ $-3x^4 - 3x^2$ $\quad\overset{\text{add to } p_0(x)}{\Rightarrow}$

$$p_1(x) = -2x^2 - x + 1$$

Step 2. Start over with $p_1(x)$ instead of $p_0(x)$:

Divide the highest degree term of $p_1(x)$ by that of $q(x)$. We need $\boxed{-2}$

multiply $q(x)$ by -2 $\quad\Rightarrow\quad$ $-2x^2 - 2$ $\quad\overset{\text{change sign}}{\Rightarrow}\quad$ $2x^2 + 2$ $\quad\overset{\text{add to } p_1(x)}{\Rightarrow}$

$$p_2(x) = -x + 3$$

Since it is impossible to continue with another step, we stop here and declare that

$$p_0(x) = (3x^2 - 2)q(x) + (-x + 3)$$