The First Fundamental Theorem of Integral Calculus
Section 6.4

First fundamental Theorem of Integral Calculus. If F is any antiderivative of the function f on the iterval $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Convention. We usually use the notation $\Big[F(x)\Big]_{x=a}^{x=b}$ to refer to the difference $F(b) - F(a)$. The following example shows how to use this new notation:

Example. Evaluate the definite integral $\displaystyle\int_{-2}^{4} \sqrt{x+4}\,dx$

Solution. Using the change of variable $u = x+4$ we convert the associated indefinite integral to

$$u = x + 4 \quad \Rightarrow \quad du = dx$$
$$\int \sqrt{x+4}\,dx = \int \sqrt{u}du = \int u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(x+4)^{\frac{3}{2}} + C$$

Since any of the antiderivatives suffices for the evaluation of the definite integral , we take the case $C = 0$, i.e. we take the antiderivative $\frac{2}{3}(x+4)^{\frac{3}{2}}$.

Then

$\int_{-2}^{4} \sqrt{x+4}\, dx = \left[\frac{2}{3}(x+4)^{\frac{3}{2}}\right]_{x=-2}^{x=4} = \frac{2}{3}\left[(x+4)^{\frac{3}{2}}\right]_{x=-2}^{x=4}$

$= \frac{2}{3}\left[16\sqrt{2} - 2\sqrt{2}\right] = (\frac{2}{3})(14\sqrt{2}) = \frac{28\sqrt{2}}{3}$

<u>Note</u>. Since the evaluation of definite integrals roots back to the evaluation of indefinite integrals , and since the indefinite integral preserve addition and scalar multiplication , we have similar properties for the definite integrals:

<u>Theorem</u>. If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x) + g(x)] dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$$

$$\int_a^b k f(x)\, dx = k \int_a^b f(x)\, dx \qquad k \text{ being a constant}$$