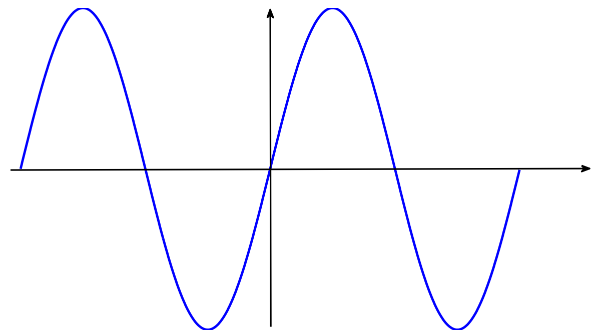
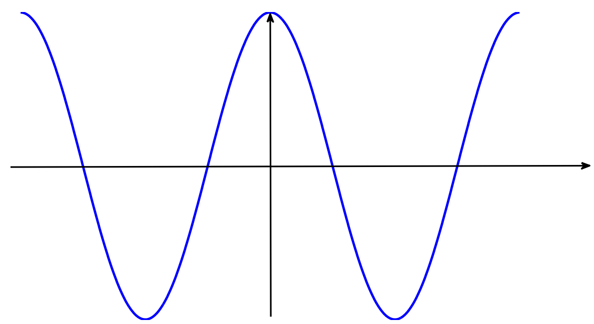
Continuity of trigonometric functions

The function $\sin(x)$ is continuous everywhere.



The function $\cos(x)$ is continuous everywhere.



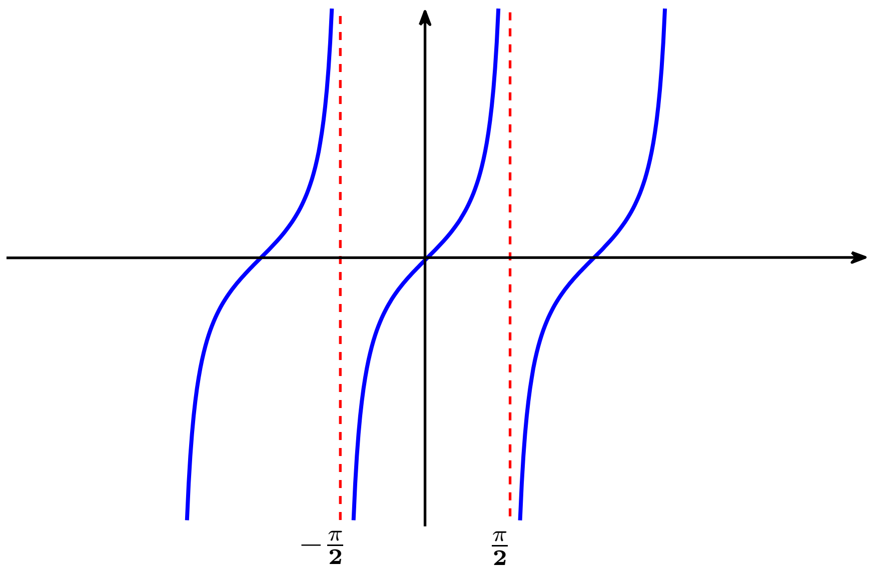
The function $y = \tan(x)$ has the set

$$D_{\tan}\left\{x \ : \ x \neq \frac{(2k+1)\pi}{2} \quad k = 0\,,\ \pm 1\,,\ \pm 2\,,\ .....\right\}$$

as its domain. Although the graph of this function has breaks at the points $\frac{(2k+1)\pi}{2}$ but this function is continuous on its domain because the points $\frac{(2k+1)\pi}{2}$ are not in its domain. The lines $x = \frac{(2k+1)\pi}{2}$ are the vertical asymptotes of the function $\tan(x)$ Also note that

$$\lim_{x \to \left(\frac{(2k+1)\pi}{2}\right)^-} \tan(x) = \infty$$

$$\lim_{x \to \left(\frac{(2k+1)\pi}{2}\right)^+} \tan(x) = -\infty$$



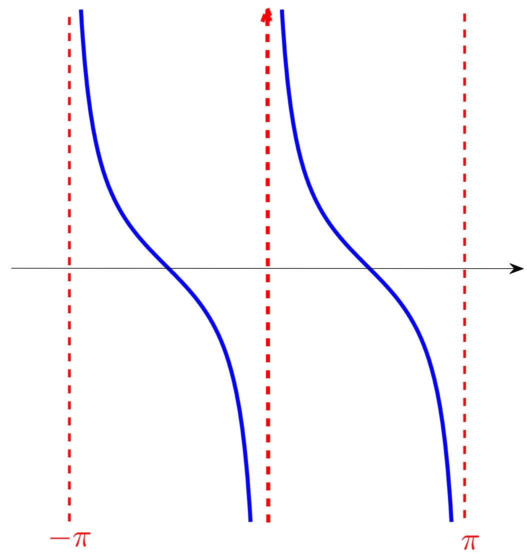
The function $y = \cot(x)$ has the set $\left\{ x \quad x \neq k\pi \quad k = 0\,, \pm 1\,, \pm 2\,, \,..... \right\}$ as its domain. Although the graph of this function has breaks at the points $k\pi$ but this function is continuous on its domain because the points $k\pi$ are not in its domain. The lines $x = k\pi$ are the vertical asymptotes of the function $\cot(x)$ Also note that

$$\lim_{x \to (k\pi)^-} \cot(x) = -\infty$$
$$\lim_{x \to (k\pi)^+} \cot(x) = +\infty$$

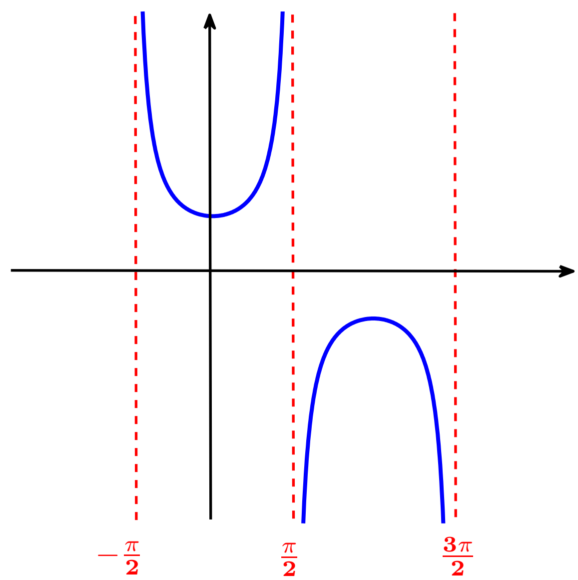
The function $y = \sec(x)$ has the set

$$\left\{ x \quad x \neq \frac{(2k+1)\pi}{2} \quad k = 0\,, \pm 1\,, \pm 2\,, \,..... \right\}$$

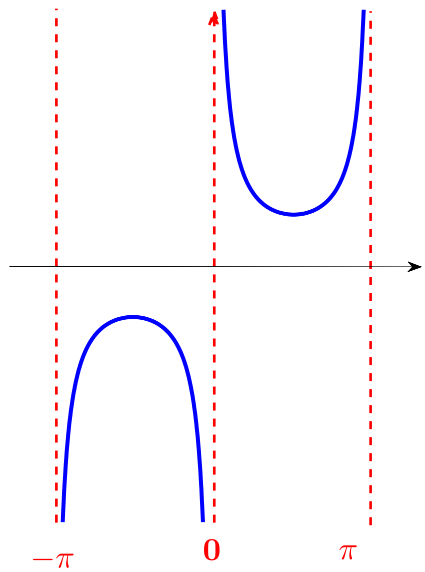
as its domain. So the domains of the functions $\tan(x)$ and $\sec(x)$ are the same. Although the graph of this function has breaks at the points $\frac{(2k+1)\pi}{2}2$ but this function is continuous on its



domain because the points $\frac{(2k+1)\pi}{2}$ are not in its domain. The lines $x = \frac{(2k+1)\pi}{2}$ are the vertical asymptotes of the function $\sec(x)$.



The function $y = \csc(x)$ has the set $\left\{x \quad x \neq k\pi \quad k = 0\,,\,\pm 1\,,\,\pm 2\,,\,.....\right\}$ as its domain. So the domains of the functions $\cot(x)$ and $\csc(x)$ are the same. Although the graph of this function has breaks at the points $k\pi$ but this function is continuous on its domain because the points $k\pi$ are not in its domain. The lines $x = k\pi$ are the vertical asymptotes of the function $\csc(x)$



Some useful inequalities:

$$|\sin x| \leq |x|$$

$$|\sin x| \leq 1$$

$$|\cos x| \leq 1$$

The inequalities $|\sin x| \leq 1$ and $|\cos x| \leq 1$ come from the fact that $\sin^2 + \cos^2 = 1$. In fact

$$\sin^2 + \cos^2 = 1 \qquad \Rightarrow \qquad \begin{cases} 0 \leq \sin^2 \leq 1 \\ 0 \leq \cos^2 \leq 1 \end{cases} \quad \overset{\text{taking square root}}{\Rightarrow} \quad \begin{cases} 0 \leq |\sin| \leq 1 \\ 0 \leq |\cos| \leq 1 \end{cases}$$