Theorem (zero-times-bounded Theorem): Let lim denote any of the limits $\lim_{x\to a}$, $\lim_{x\to a^+}$, $\lim_{x\to a^-}$, $\lim_{x\to\infty}$, and $\lim_{x\to-\infty}$. Let for the points close to the point where the limit is being calculated at the function f remains bounded. Suppose further that $\lim g(x) = 0$. Then $\lim f(x)g(x) = 0$.

Example: Evaluate

$$\lim_{x\to\infty} \frac{x^3 + x^2 \sin(x^2+1)}{x^3+1}$$

Solution:

$$= \lim_{x\to\infty} \frac{x^3\left(1+\frac{1}{x}\sin(x^2+1)\right)}{x^3\left(1+\frac{1}{x^3}\right)} = \lim_{x\to\infty} \frac{1+\frac{1}{x}\sin(x^2+1)}{1+\frac{1}{x^3}} = \frac{1+0}{1+0} = 1$$

noting that the term $\frac{1}{x}$ tends to zero while $\sin(x^2+1)$ is bounded , therefore by an application of the zero-times-bounded theorem the limit of $\frac{1}{x}\sin(x^2+1)$ is zero.