Solutions to Practice Problem

a) Model (2) allows age to have a non-linear effect, since it includes an Age^2 term. However, if the slope coefficient on Age^2 is estimated to be statistically insignificant, then we can't reject the null hypothesis that the effect of Age is linear.

H_0: effect of Age is linear (i.e. $\beta_8 = 0$)
H_A: Age has a non-linear effect (i.e. $\beta_8 \neq 0$)

We can use a t-test:

$$t = \frac{-0.0002}{0.0002} = -1$$ (use table pg. 351) p-val = 0.1587

We fail to reject the null at the 10% significance level.

b) The estimated slope coefficient on Age isn't statistically sig. However, to test if Age and Age^2 are jointly equal to zero, we need the F-test. One formula is:

$$F = \frac{(R_U^2 - R_R^2)/q}{(1-R_U^2)/(n-k_u-1)}$$

There is a problem. The above formula uses R^2, but we only have $\bar{R}^2$. We need the following formula:

$$R^2 = 1 - (1-\bar{R}^2)\frac{n-k-1}{n-1}$$

R^2 model 1 : $1 - (1-0.144)\dfrac{463-6-1}{463-1} = 0.155$

R^2 model 2 : $1 - (1-0.142)\dfrac{463-8-1}{463-1} = 0.157$

Now to state the null and alternative hypotheses :

H_0 : model (1) (i.e. $\beta_7 = \beta_8 = 0$) (restricted model)

H_A : model (2) (unrestricted model)

$$F = \frac{(0.157 - 0.155)/2}{(1-0.157)/(463-8-1)} = 0.54$$

Using the table on pg. 355, we see that our critical value at the 10% significance level is 2.30. Hence, we fail to reject the null.

c) Model (3) adds an <u>interaction term</u>, which allows the effect of Beauty to be different for males and females.

For men : estimated effect = 0.231

For women : estimated effect = 0.231 - 0.141 = 0.090

The interaction term is statistically significant at the 5% level. Omitting this term would likely cause $\hat{\beta}_1$ in model (1) and (2) to be biased.

d) Since the effect is non-linear, we need to know Age. Since Dr. Godwin is 31 years old, the predicted effect is:

$$\widehat{Course\_Eval}\,|^{Age=32} - \widehat{Course\_Eval}\,|^{Age=31}$$

$$= 0.020 - 0.0002(32^2) + 0.0002(31^2) = 0.0074$$

e) In both models, the effect is 0.231. Hence, Course_Eval should increase by 0.0231.

f) We can't add the variable "Female x Beauty" because it is exactly linearly related to the variable "Male x Beauty." The proposed model would suffer from perfect multicollinearity, or the "Dummy variable trap." The regression software will be unable to estimate the model.

g) We could possibly drop the variables "Intro" and "Minority" since they are individually statistically insignificant. However, to drop both of them at once, we would need an F-test.