Midterm Answer Key   ECON 3180 - 2013

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1	2.5	3.5	4.5	5.5
2	B	C	D	A
3	D	A	B	C
4	B	B	B	B
5	C	D	A	B
6	D	A	B	C
7	D	A	B	C
8	C	D	A	B

9. $\bar{Y} = \frac{1+4+7}{3} = 4$

$$s_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{3-1}\left[(1-4)^2 + (4-4)^2 + (7-4)^2\right] = \frac{18}{2} = 9$$

(4 marks)

We divide by "2" instead of "3" so that s_Y^2 is an unbiased estimator of σ_Y^2.

-OR-

Dividing by "2" instead of "3" is a degrees-of-freedom correction. We lose one d.o.f. when we calculate $\bar{Y}$, which is needed to calculate s_Y^2.

(2 marks)

10. If $\hat{\beta}_1$ is unbiased, then $E(\hat{\beta}_1 | X=x) = \beta_1$. This says that, regardless of what the X variable is, $\hat{\beta}_1$ gives us the "right answer" on average. That is, we expect our estimator ($\hat{\beta}_1$) to be the true value (β_1) on average.

In the provided formula, the only way that $E(\hat{\beta}_1)$ equals the true β is if the very last term disappears (is zero). This happens if $E(u_i | X=x) = 0$.

11. The predicted value, $\hat{Y}_i$, when $X_i = 4$ is:

$$\hat{Y}_i = 41.2 + 5.2(4) = 62$$

The OLS residual, $\hat{u}_i$, can't be found, because we don't know Y_i.

12. When the sample size becomes very large, the variance of $\bar{Y}$ becomes zero. That is,

$$\lim_{n \to \infty} \frac{\sigma_Y^2}{n} = 0$$

Since $\bar{Y}$ is unbiased $(E(\bar{Y}) = \mu_Y)$, $\bar{Y}$ will give us the "right answer" when n is very large. This property is called <u>consistency</u>.

Q.13)

a) The population model is:

$$Wage = \beta_0 + \beta_1 \times UNI + u_i$$

The 'dummy' variable, UNI, equals 1 if the person has a university education. The expected wage, or mean wage of someone who has a university education is therefore:

$$E(Wage \mid UNI = 1) = \beta_0 + \beta_1 \qquad \text{(under L.S.A. \#1)}$$

The estimated mean wage, from the sample, is $\hat{\beta}_0 + \hat{\beta}_1$.

$$\hat{\beta}_0 + \hat{\beta}_1 = 10.24 + 8.52 = \underline{\underline{18.76}}$$

b) The estimated mean wage for workers without a university education is $\hat{\beta}_0$ (10.24). The effect, on mean wage, of obtaining a university degree is β_1, which has been estimated at 8.52.
In order to answer the question, we must conduct a hypothesis test to determine whether or not $\hat{\beta}_1$ is statistically different from zero.

$$H_0: \beta_1 = 0 \quad ; \quad H_A: \beta_1 \neq 0$$

$$t\text{-stat} = \frac{\hat{\beta}_1 - \beta_{1,0}}{s.e.(\hat{\beta}_1)} = \frac{8.52}{4.20} = 2.03$$

[This has been stated as a two-sided alternative. You may use a one-sided alternative, but must interpret your p-values correctly]

We will assume that 'n' is large enough, so that $t\text{-stat} \sim N(0,1)$.
Using the tables provided, we find that the one-sided p-value is 0.0212, so that the 2-sided p-value is 0.0424.

Based on this p-value, we reject the null hypothesis at the 10% and 5% significance levels, but not at the 1% significance level.

We find evidence that supports the idea that obtaining a university degree increases the mean wage of workers.

c) Heteroskedasticity will arise if there is more variation in 'Wage' amongst one of the groups. This is likely to be the case. While obtaining a university degree does not ensure a worker will make more than minimum wage, a worker without a degree is likely prohibited from receiving a high wage (as discussed in class). Since there are more possibilities (more spread) for degree-holders, variance should be higher among this group.

The problem is that if we don't account for heteroskedasticity, we will be using the wrong formula to estimate the standard error of $\hat{\beta}_1$.

Heteroskedasticity will not affect our answers in part (a). Only $\hat{s.e.}(\hat{\beta}_1)$ is effected. $\hat{\beta}_1$ is still unbiased and consistent. (Same for $\hat{\beta}_0$). However, our answers to part (b) might be wrong. With a different value for $\hat{s.e.}(\hat{\beta}_1)$, we will calculate a different t-stat, and a different p-value.

d) From the reported estimated equation, we know that the estimated mean wage of a worker who has a university education is :

$$\widehat{E(Wage \mid UNI=1)} = \hat{\beta}_0 + \hat{\beta}_1 = 10.24 + 8.52 = 18.76$$

Under the new model, where UNI has been defined in the opposite way, the mean wage for degree holders is

$$E(Wage \mid UNI = 0) = \beta_0$$

Since nothing in the sample has changed, it must be that:

$$\hat{\beta}_0 = 18.76$$