Econ 3180 - Midterm 2015 Answer Key

The exam version is determined by how the last word of the exam ("end") is written.

Q	Version: END.	END	end	end.
1.	C	A	B	D
2.	A	B	C	D
3.	C	A	B	B
4.	B	D	A	C
5.	B	C	D	A
6.	D	A	B	C

7.

$R^2 = 0$

R^2 will equal zero when $\hat{\beta}_1 = 0$. This is a situation of "no fit." In this case, variation in X does not explain any of the variation in Y.

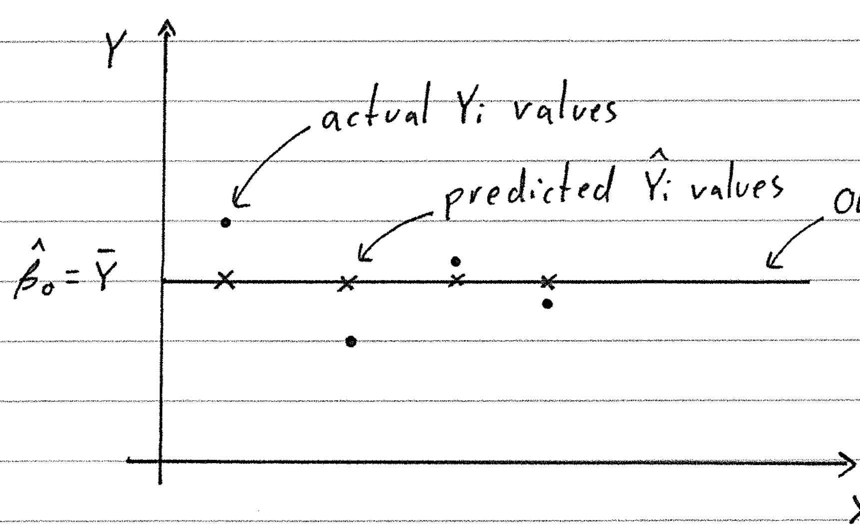
When $\hat{\beta}_1 = 0$, the OLS predicted values will be equal to $\hat{\beta}_0$. That is, $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = \hat{\beta}_0$.

The equation for the OLS estimator of the intercept is:

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}.$$

Hence, if $\hat{\beta}_1 = 0$, $\hat{Y}_i = \hat{\beta}_0 = \bar{Y}$. That is, all of the OLS predicted values are just equal to the sample average, $\bar{Y}$.

In this case, there is no variation in the predicted values, so $ESS = 0$, and $R^2 = 0$.



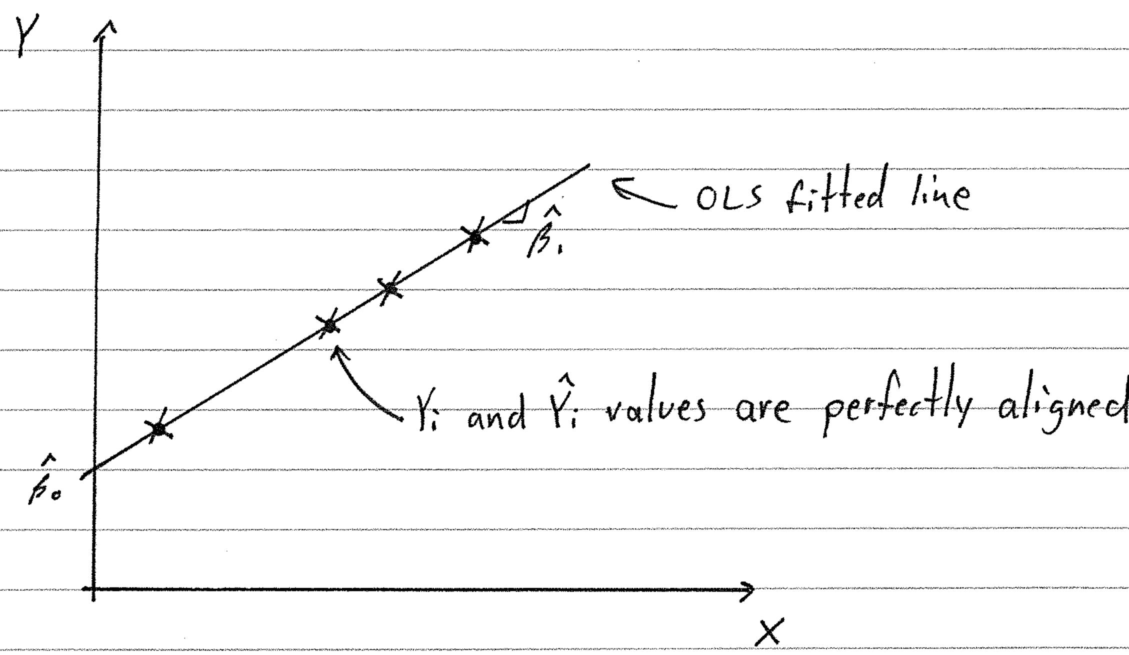
7.

$R^2 = 1$

R^2 will equal one when all of the data points lie on a straight line. This is a situation of "perfect fit." In this case, variation in X explains all of the variation in Y.

In this case, all of the OLS predicted values will be exactly equal to the corresponding actual value. That is, $\hat{Y}_i = Y_i \;\; \forall i$.

In this case, all of the OLS residuals are zero, and RSS = 0. Also, ESS = TSS, so $R^2 = 1$.



8. See the answer Key for "Mid-term Practise," Q1.

9a) β_1 is the slope of the population model. It determines how much TestScore will change given a one unit increase in STR. In economics terms, this is called a marginal effect.

b) β_0 is the intercept of the population model. It is the value that TestScores would take if STR=0. Since there can not be a class with zero students, β_0 has no economic meaning or interpretation.

c) u_i is the random error term in the population model. It contains all of the other factors that effect TestScore except STR. There were many examples given in class and in the textbook, of factors that may be inside of u_i in this ~~er~~ model.

10. a) The R^2 of 0.919 means that the variable cheese explains 91.9% of the variation in civil.

b) The answer to this question depends on the version. For example, if per-capita consumption of cheese is predicted to be 10.6, then the estimated model would predict the number of degrees awarded to be:

$$\widehat{civil} = -988.15 + 157.11(10.6) = 677.22$$

c) The 95% C.I. around $\hat{\beta}_0$ is:

$$\hat{\beta}_0 \pm 1.96 \times s.e.(\hat{\beta}_0)$$

$$= -988.15 \pm 1.96(167.10)$$

$$= [-1315.7 \, , -660.6]$$

d) The null and alternative hypothesis can be written as:

$$H_0: \beta_1 = 0 \quad , \quad H_1: \beta_1 \neq 0$$

The test-statistic is:

$$t = \frac{\hat{\beta}_1^{ACT} - \beta_{1,0}}{s.e.(\hat{\beta}_1)} = \frac{157.11 - 0}{16.49} = 9.527$$

This is a very large t-statistic (the largest in the table was 3.4!) The p-value associated with this t-stat will be very small. We should reject the null at any significance level.

e) The R^2 indicates that the two variables are correlated. The scatter-plot shows there is a direct relationship. The t-test indicates that $\beta_1 \neq 0$.
However, these are all just measures of correlation, and correlation does not imply causation!
I could estimate the model:

$$cheese_i = \beta_0 + \beta_1 civil_i + u_i,$$

and again, $\hat{\beta}_1$ would be significant. It does not mean that civil causes cheese.

The population model is silly, cheese consumption should not cause civil engineering PhD's to be awarded. However, since the two variables are correlated, there is no <u>statistical</u> way to refute this causality.