ECON 7010

Assignment #1

Use the following data for Question 1:

$$Y = \{0, 5, 6, 3\}$$

$$X = \{2, 4, 6, 8\}$$

where Y is the dependent variable, and X is the independent variable. The population model is:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

1.1) Write R code which calculates b_0 and b_1. The R code should use basic matrix-algebra functions only. Do not use the R functions *mean*, *var*, *cov*, or *lm*.

The basic functions which you might use are:

- combine http://stat.ethz.ch/R-manual/R-devel/library/base/html/c.html
- matrix http://stat.ethz.ch/R-manual/R-devel/library/base/html/matrix.html
- solve http://stat.ethz.ch/R-manual/R-devel/library/base/html/solve.html
- transpose http://stat.ethz.ch/R-manual/R-devel/library/base/html/t.html
- matrix multiplication http://stat.ethz.ch/R-manual/R-patched/library/base/html/matmult.html

For extracting elements from a matrix see: http://www.r-tutor.com/r-introduction/matrix. This short tutorial also illustrates the *matrix* command.

For a quick list of matrix commands see: http://www.statmethods.net/advstats/matrix.html

1.2) Write R code to calculate the predicted values, and the residuals, from the estimated model above.

1.3) Write R code to calculate the R^2 from the estimated model above.

1.4) Use the R function *lm* to verify your answers above.

2) Prove that $\bar{y} = \bar{\hat{y}}$, if the model includes an intercept.

3) Suppose that we have the following population model:

$y = X\beta + \varepsilon,$

where the X matrix contains only a column of "ones". In this case, prove that $b = \bar{y}$.

4) Suppose that we have a linear multiple regression model,

$$y = X\beta + \varepsilon$$

$$= X_1\beta_1 + X_2\beta_2 + \varepsilon \ ,$$

where all of the usual assumptions are satisfied, except that $E(\varepsilon) = X_1\gamma$. That is, the mean vector for the disturbances is a linear combination of a subset of the regressors.

Let b_1 and b_2 be the OLS estimators of β_1 and β_2. Obtain the expressions for $E(b_1)$ and $E(b_2)$, and interpret your results.

5) Derive the variance of the OLS estimator.