Assignment 1 Answer Key

2. See Topic 1, pg. 11.

3. The general formula for the OLS estimator is:

$$b = (X'X)^{-1}X'y$$

In the case of a population model where:

$$y_i = \beta_0 + \varepsilon_i,$$

i.e. where the model contains only an intercept, the X matrix consists only of a column of 1^s. Hence,

$$b = \left([1 1 \cdots 1] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \right)^{-1} [1 1 \cdots 1] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$= \frac{1}{n} \quad \Sigma y_i = \bar{y}$$

4. The OLS estimator for β_1 is:

$$b_1 = (X_1' M_2 X_1)^{-1} X_1' M_2 y \,, \quad \text{where } M_2 = I - X_2 (X_2' X_2)^{-1} X_2'$$

Taking the expectation of b_1, and substituting in the pop. model for y, we have:

$$E(b_1) = E\left[(X_1' M_2 X_1)^{-1} X_1' M_2 (X_1 \beta_1 + X_2 \beta_2 + \varepsilon) \right]$$

Using A.5, and expanding the expression:

$$E(b_1) = (X_1' M_2 X_1)^{-1} X_1' M_2 X_1 \beta_1 + (X_1' M_2 X_1)^{-1} X_1' M_2 X_2 \beta_2$$
$$+ (X_1' M_2 X_1)^{-1} X_1' M_2 E(\varepsilon)$$

Using $E(\varepsilon) = X_1 \gamma$, $M_2 X_2 = 0$, and simplifying, we get:

$$E(b_1) = \beta_1 + 0 + \gamma$$

We will get a similar result for $E(b_2)$, except that the third term will cancel:

$$E(b_2) = 0 + \beta_2 + (X_2' M_1 X_2)^{-1} X_2' M_1 X_1 \gamma = \beta_2$$

5. See Topic 1 (continued), pg. 22-23.