

ASSIGNMENT #2, Q3 ANSWER KEY

The fitted model is:

$$y = \beta_1 + \beta_2 X + \varepsilon. \quad (1)$$

The true model is:

$$y = \beta_2 X + u. \quad (2)$$

In the fitted model, we have failed to impose a restriction ($\beta_1 = 0$).

$$\text{In (1): } V(b) = \sigma^2 (X'X)^{-1} = \frac{\sigma^2}{n \sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}$$

$$\text{So, } \text{var}(b_2) = V(b)_{22} = \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2} \quad (3)$$

$$\text{In (2): } \text{var}(b_2) = \frac{\sigma^2}{\sum x_i^2} \quad (4)$$

Hence, (3) > (4).

In this example, including an unnecessary regressor (the intercept) has caused the variance of b_2 to increase.