Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Fall 2014

Assignment #3

Question 1:
Consider the usual linear multiple regression model:

$$y = X\beta + \varepsilon; \qquad \varepsilon \sim N(0, \sigma^2 I)$$

Suppose that we decide to use the following estimator for β: $\hat{\beta} = [A + X'X]^{-1}X'y$, where A is a positive definite, symmetric, ***non-random matrix with known elements***.

(a) Is $\hat{\beta}$ a linear estimator of β, or a non-linear estimator?
(b) Show that this estimator weakly consistent?
(c) Show that $\hat{\beta}$ is a biased estimator of β. What is the expression for its bias?
(d) Now form a new 'bias-adjusted estimator' by subtracting the bias expression from $\hat{\beta}$. Why can't this 'estimator' actually be applied in practice?
(e) Use the OLS estimator of β to modify this 'bias-adjusted' estimator so that it *can* be used. What is this final estimator that you have constructed?

Question 2:
When we establish the (weak) consistency of the OLS estimator for the regression coefficient vector, we usually assume that

$$p\lim[\frac{1}{n}X'X] = Q; \qquad Q \text{ is a finite, positive-definite matrix.}$$

(1)
Suppose that we have a model with just <u>two</u> regressors – an intercept and a linear time-trend variable (a variable that takes the values 1, 2, 3,….., n).

(a) Is Q finite and positive-definite in this case?
(b) Obtain the expressions for the mean and variance of the OLS slope coefficient estimator for this special model.
(c) Use these expressions to prove that this OLS estimator is 'mean square consistent'.
(d) Is this estimator 'weakly consistent'?
(e) Is the OLS estimator of the intercept weakly consistent for this model?
(f) So, what do you conclude about the role of the usual assumption, (1)?

[<u>Hint</u>: The sum of the first 'n' integers is $n(n + 1) / 2$. The sum of the squares of the first 'n' integers is $n(n + 1)(2n + 1) / 6$.]

Question 3:

Consider the linear multiple regression model, with k non-random regressors, including an intercept:

$$y = X\beta + \varepsilon \;;\; \varepsilon \sim N[0, \sigma^2 I] .$$

Let 'F' denote the F-statistic for testing the hypothesis that all of the slope coefficients (*i.e.*, all of the coefficients except that for the intercept) are zero.

(a) Prove the following relationship:

$$F = [R^2/(k-1)]/[(1-R^2)/(n-k)] .$$

(b) Use this result to show that if R^2 increases (decreases), then F must increase (decrease).