Heteroskedasticity and GLS practice questions

Q1:
Suppose that we have a standard linear multiple regression model, with k regressors:

$$y = X\beta + \varepsilon,$$

where the error term and regressors satisfy all of the usual assumptions, except that the covariance matrix of the error vector is Σ. Suppose that $\Sigma X = X\Theta$, for some $(k \times k)$ nonsingular matrix Θ.

Prove that in this model the OLS and GLS estimators β have the same covariance matrix in this case.

Q2:
Critically appraise each of the following statements relating to heteroskedasticity in a regression model:

(a) "In the presence of heteroskedasticity the OLS estimator of the coefficient vector is inefficient and this causes the usual t tests and F tests to be invalid."

(b) "The OLS estimator is unbiased but it is inconsistent if the errors are heteroskedastic."

(c) "One way of eliminating the problems associated with heteroskedasticity is to transform the model so that the errors are normally distributed and uncorrelated with the regressors."

(d) "Sometimes apparent heteroskedasticity can be caused by a mathematical misspecification of the regression model. This can happen, for example, if the dependent variable ought to be logarithmic, but a linear regression is estimated."