Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Midterm, Oct. 08, 2015

Instructor: **Ryan Godwin**
Instructions: **Answer ALL QUESTIONS, and put all answers in the booklet provided**
Time Allowed: **75 minutes (Total marks = 75 – *i.e., one mark per minute*)**
Number of Pages: **3**

PART A:

Select the most appropriate answer in each case. Each question is worth ***3 marks****. (No explanation is needed to obtain full marks, but it will be taken into account if given.)*

1.) If we use Ordinary Least Squares to estimate the regression model, $y = X\beta + \varepsilon$, where *all* of the usual assumptions are satisfied, then:

a) The OLS residuals will sum to zero.
b) The regressors will be correlated with the residuals in the sample.
c) Adding any regressor will not increase the sum of squared residuals.
d) All of the above.

2.) The Gauss-Markov Theorem tells us that, under appropriate assumptions about the model:

a) The least squares estimator of β in the usual linear regression model has the smallest possible variance, and therefore is 'best'.
b) The least squares estimator of β in the usual linear regression model has the smallest mean squared error among all possible linear estimators for this parameter vector.
c) The least squares estimator of β in the usual linear regression model has the smallest mean squared error among all possible unbiased estimators of this parameter vector.
d) The least squares estimator of β in the usual linear regression model is most efficient among all possible linear and unbiased estimators of this parameter vector.

3.) The Least Squares principle for estimating a regression model, $y = X\beta + \varepsilon$, where *all* of the usual assumptions are satisfied:

a) Produces unbiased and efficient estimators of both β and σ.
b) Involves minimizing the sum of the squares of elements of ε.
c) Produces an estimator for β that has a Normal sampling distribution, centered at β itself.
d) Produces an equal number of positive and negative residuals if the sample size is even.

4.) A p-value is:

a) The probability of calculating a test statistic more extreme than the one just calculated.
b) The maximum and minimum values for the test statistic, that won't be rejected in a hypothesis test.
c) The maximum and minimum values for the null hypothesis, that won't be rejected in a hypothesis test.
d) Equal to the probability of a type I error.

5.) The correct interpretation of a 95% confidence interval constructed around a regression coefficient is:

a) There's a 0.95 probability that the true value of the regression coefficient lies in the interval.
b) The interval includes the true value of the regression coefficient 95% of the time.
c) 95% of such intervals will contain the true value of the regression coefficient.
d) None of the above.

PART B: *Answer all questions.*

6.) Prove that the OLS residuals will be Normally distributed if the error term is Normally distributed.

[10 marks]

7.) Describe (briefly) a situation where:

a) $R^2 = 0$

b) $R^2 = 1$

[10 marks]

8.) Suppose that the true population model is: $y = X_1\beta_1 + X_2\beta_2 + u$, but the model that you estimate by OLS is $y = X_1\beta_1 + \varepsilon$.

a) Show that, in general, the OLS estimator for β_1 is biased.

[10 marks]

b) Under what two conditions will the OLS estimator for β_1 be unbiased?

[5 marks]

9.) Show how to express the quantity:

$$\sum_{i=1}^{n}(y_i - \bar{y})^2,$$

in matrix form.

[5 marks]

10.) Let $\tilde{\beta} = (X^{*\prime}X^*)^{-1}X^{*\prime}y^*$, where $X^* = CX$ and $y^* = Cy$. C is an $n \times n$ matrix, where the first $^n/_2$ leading diagonal elements of C are equal to “1”, and the rest of the elements are “0”. For example, if $n = 6$, then:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

a) Prove that $\tilde{\beta}$ is an unbiased estimator for β, in the population model $y = X\beta + \varepsilon$.

[10 marks]

b) Intuitively, explain why you would expect $\tilde{\beta}$ to be unbiased.

[5 marks]

c) We know from the Gauss-Markov theorem that $V(\tilde{\beta}) - V(b)$ is positive definite, where b is the OLS estimator. Intuitively, explain why we would expect $V(\tilde{\beta}) - V(b)$ to be positive definite, even if we did not know the Gauss-Markov theorem.

[5 marks]

END.