Midterm Solutions

Part A - Multiple Choice

1. A   2. D   3. D   4. B   5. C   6. D

Part B - Short Answer

7. False

It will pass through the mean if the model includes an intercept. So, the sum of the positive residuals equals the sum of the negative ones - but that's all we can say. Nothing is implied about the number of positive and negative residuals.

8. False

A large p-value implies we cannot reject H_0 (that the regressors and errors are uncorrelated). So we would use OLS, not I.V. estimation.

9. False

The asymptotic distribution of b_{OLS} is: $\sqrt{n}(b_{OLS}-\beta) \xrightarrow{d} N[0, \sigma^2 Q^{-1}]$, while the asymptotic distribution of b_{IV} is:
$\sqrt{n}(b_{IV}-\beta) \xrightarrow{d} N(0, \sigma^2 Q_{ZX}^{-1} Q_{ZZ} Q_{XZ}^{-1})$. Since it can be shown that $Q_{ZX}^{-1} Q_{ZZ} Q_{XZ}^{-1} - Q^{-1}$ is positive-semi-definite, we either lose (or maintain) efficiency when we use b_{IV} instead of b_{OLS}.

10. False

If the restriction is true ($q = R\beta$) then:

$$E(Rb - R\beta) = R\beta - R\beta = 0,$$

and $Rb - q$ should be small. The Wald test statistic will not appear "extreme" when compared to the Chi-Square distribution, and we will fail to reject the null.

Part C – Long Answer

11. a) $b_1 = (X_1' M_2 X_1)^{-1} X_1 M_2 y$, where $M_2 = I - X_2(X_2'X_2)^{-1}X_2'$

$$= (X_1' M_2 X_1)^{-1} X_1' M_2 (X_1\beta_1 + \varepsilon) = \beta_1 + (X_1' M_2 X_1)^{-1} X_1 M_2 \varepsilon$$

$$E(b_1) = \beta_1 + 0 = \beta_1 \quad \text{unbiased}$$

$$b_1 = \beta_1 + (X_1'(I - X_2(X_2'X_2)^{-1}X_2')X_1)^{-1} X_1'(I - X_2(X_2'X_2)^{-1}X_2')\varepsilon$$

$$= \beta_1 + (X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1}(X_1'\varepsilon - X_1'X_2(X_2'X_2)^{-1}X_2'\varepsilon)$$

$$= \beta_1 + \left(\frac{X_1'X_1}{n} - \frac{X_1'X_2}{n}\left(\frac{X_2'X_2}{n}\right)^{-1}\frac{X_2'X_1}{n}\right)^{-1}\left(\frac{X_1'\varepsilon}{n} - \frac{X_1'X_2}{n}\left(\frac{X_2'X_2}{n}\right)^{-1}\frac{X_2'\varepsilon}{n}\right)$$

$$\text{plim}(b_1) = \beta_1 + (Q_{11} - Q_{12}Q_{22}^{-1}Q_{21})^{-1}(0 - Q_{12}(Q_{22})^{-1}0) = \beta_1 \quad \text{consistent}$$

$$b_2 = (X_2' M_1 X_2)^{-1} X_2' M_1 y = \beta_2 + (X_2' M_1 X_2)^{-1} X_2 M_1 \varepsilon$$

$$E(b_2) = \beta_2 + 0 = \beta_2 \quad \text{unbiased}$$

by symmetry: $\text{plim}(b_2) = \beta_2 + (Q_{22} - Q_{21}Q_{11}^{-1}Q_{12})^{-1}(0 - Q_{21}Q_{11}^{-1}0) = \beta_2$

b) $b_1 = (X_1'X_1)^{-1}X_1'(X_1\beta_1 + X_2\beta_2 + \varepsilon) = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\beta_2 + (X_1'X_1)^{-1}X_1'\varepsilon$

$$E(b_1) = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\beta_2 + 0 \quad \text{biased}$$

$$\text{plim}(b_1) = \beta_1 + Q_{11}^{-1}Q_{12}\beta_2 + Q_{11}^{-1}0 \neq \beta_1 \quad \text{inconsistent}$$

Note: part (a) may also be answered using the "S" matrix defined in class.

c) The OLS estimator is derived by minimizing $e'e$. When a regressor is added, a restriction is relaxed, so $e'e$ must decrease or stay the same.

Since $R^2 = 1 - \frac{e'e}{y'M^0y}$, $R^2 \uparrow$ when $e'e \downarrow$.

d) The cost of excluding a relevant regressor outweighs the cost of including an irrelevant one. In the former, we lose consistency. In the latter, we only lose efficiency. This suggests that we should start with a general model and carefilly consider eliminating regressors.

12. a) $E[s^2] = \sigma^2$, $E\left[\frac{e'e}{n-k}\right] = \sigma^2$, $E[e'e] = (n-k)\sigma^2$

$$E\left[\frac{e'e}{n-c}\right] = \frac{(n-k)\sigma^2}{n-c}$$

$$\text{Bias}(\tilde{\sigma}^2) = \frac{(n-k)\sigma^2}{n-c} - \sigma^2 = \sigma^2\left(\frac{n-k}{n-c} - 1\right)$$

if $k > c$, Bias is $(-)$, etc.

b) From the hint: $\text{var}\left[\frac{(n-k)s^2}{\sigma^2}\right] = 2(n-k)$

So, $\text{var}[s^2] = \text{var}\left[\frac{e'e}{n-k}\right] = \frac{2\sigma^4}{n-k}$

$\text{var}[e'e] = 2\sigma^4(n-k)$

Therefore, $\text{var}\left[\frac{e'e}{n-c}\right] = \frac{2\sigma^4(n-k)}{(n-c)^2}$

c) As $n \to \infty$, Bias $\to 0$ and var $\to 0$.
So, we have strong consistency (mean-squar consistency), which implies weak consistency.

13. a) $t_4 = \frac{b_4 - 0}{0.070}$, $\frac{PNC}{PD} = t_4 \cdot se(b_4) = -4.647 \cdot 0.070$
$= -0.326$

$t_y = \frac{9.881}{0.904} = 10.934$

b) $s^2 = \frac{e'e}{n-k} \Rightarrow e'e = s^2(n-k) = (0.030^2)(31) = 0.029$

c) $[-0.082 - (2.75)(0.013),\ -0.082 + (2.75)(0.013)] =$
$[-0.1165, -0.0477]$

d.o.f. $= n-k = 36-5 = 31$

d) $H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$
H_A: not H_0.

This F-statistic is large. We are likely to reject the null.