Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Midterm, Oct. 21, 2013

Instructor: **Ryan Godwin**
Instructions: **Answer ALL QUESTIONS, and put all answers in the booklet provided**
Time Allowed: **100 minutes (Total marks = 100 – *i.e., one mark per minute*)**
Number of Pages: **4**

PART A:
*Select the most appropriate answer in each case. Each question is worth **4 marks**. (No explanation is needed to obtain full marks, but it will be taken into account if given.)*

1. The formula for the OLS estimator *b* is derived by:

a) choosing a linear estimator which is unbiased, efficient, and consistent.
b) solving an optimization problem.
c) minimizing the distance between the fitted line and each data point.
d) econometrics, the darkest of the economics magics.

2. The probability of a 'Type I' error for any statistical test:

(a) Is just one minus the power of the test.
(b) Is equal to one minus the probability of the 'Type II' error for the test.
(c) Will always be less that the probability of the 'Type II' error for the test.
(d) Is chosen in advance by the researcher, who then tries to design the test so as to make the test as powerful as possible.

3. When we construct a confidence interval for one of the coefficients in a regression model:

(a) This interval will generally be wider if the sample size is increased, other things equal.
(b) This interval will be shorter if the confidence level is increased, other things being equal.
(c) The interval will be shorter if the standard error for the estimated coefficient is larger, other things being equal.
(d) None of the above.

4. In classical hypothesis testing, we:

(a) determine the distribution of the test statistic when the null hypothesis is false.
(b) reject the null hypothesis when the value we obtain for the test statistic seems "rare".
(c) know that the test statistic is normally distributed due to the central limit theorem.
(d) choose the power of the test in advance.

5. The Gauss-Markov Theorem tells us that, under appropriate assumptions, the least squares estimator of β in the usual linear regression model:
(a) Is a linear estimator, and therefore is "best".
(b) Has the smallest bias among all possible linear estimators for this parameter vector.
(c) Is most efficient among all possible linear and unbiased estimators of this parameter.
(d) Is most efficient among all possible unbiased estimators that have a Normal sampling distribution.

PART B:

State whether each of the following is TRUE or FALSE, and BRIEFLY explain your answer. Each question is worth ***6 marks****. Of these,* ***5 marks*** *are given for the explanation.*

6. Consider the population model:

$$y = X\beta + \varepsilon,$$

where X includes an intercept. Suppose you want to know the predicted value, $\hat{y}$ when $X = \bar{X}$, and when the model is estimated according to the least squares principle. You do not need the X data in order to calculate $\hat{y}$.

7. In general, the cost of including an irrelevant regressor is greater than the cost of excluding a relevant one.

8. The goodness-of-fit measure, R^2, must increase when a variable is added to the regression model.

9. The OLS residuals,$\boldsymbol{e}$, are consistent estimators for the error terms, $\boldsymbol{\varepsilon}$.

10. Suppose that you try to fit the following model by OLS:

$$earnings = \beta_0 + \beta_1 M + \beta_2 F + \beta_3 age + \cdots + \varepsilon,$$

where $M = 1$ if the individual is male and equals zero otherwise, and where $F = 1$ if the individual is female and equals zero otherwise. In this case, assumption A.5 of the classical linear regression model is violated: the regressors are correlated with the random errors, and this model can't be estimated by OLS.

11. To establish the weak consistency of the OLS estimator, we require the assumption that: $plim\left(\frac{X'X}{n}\right) = Q$.

PART C: *Answer all questions.*

12. Suppose that the true data-generating process is

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

where the regressors and the error term satisfy all of the usual assumptions. However, we fit the following model to the data by OLS:

$$y = X_1\beta_1 + X_3\beta_3 + u$$

(a) Do you think the OLS estimators for β_1 and β_3 are biased or unbiased in this situation? Show as much math as you can.

[8 marks]

(b) For the estimators in part (a) in which you found bias: under what special circumstance will the estimator(s) be unbiased?

[3 marks]

(c) What can you say about the efficiency of the OLS estimator, as compared to what it would be if we fitted the 'correct' model?

[3 marks]

13. Suppose that we have a linear multiple regression model,

$$y = X\beta + \varepsilon$$

where the regressors may be random and may be correlated with the errors, even asymptotically. Accordingly, we decide to use a (generalized) I.V. estimator, where Z is the (n x L) matrix of (possibly random) instruments. Prove, algebraically, that the I.V. estimator is equivalent to the following two-step estimator:

(i) Regress X on Z by OLS, and get the predicted matrix, $\hat{X}$.

(ii) Fit the following artificial model by OLS: $y = \hat{X}\beta + u$.

[8 marks]

14. Suppose that $\tilde{\theta}$ is an intuitive way to estimate the population parameter θ, however, $\tilde{\theta}$ is biased. The expected value of $\tilde{\theta}$ is:

$$E(\tilde{\theta}) = \frac{m-j}{m}\theta,$$

where m and j are known constants.

(a) Using $\tilde{\theta}$, construct a new estimator which is unbiased.

[4 marks]

(b) Provide an example of where the above strategy has been employed.

[3 marks]

15. The following variables are relevant for this question:

WW - Women's wages (hourly, in constant 1975 dollars)
WA - Women's age (years)
WE - Women's educational achievement (years)

The following is an estimated regression model that explains log(WW) in terms of log(WA) and log(WE).

Dependent Variable: LOG(WW)
Method: Least Squares
Date: 10/03/08 Time: 10:19
Sample: 1 428
Included observations: 428

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.213652	0.825244	-3.894183	0.0001
LOG(WA)	0.317758	0.178941		0.0765
LOG(WE)	1.277807	0.175543	7.279171	0.0000

R-squared	0.113858	Mean dependent var	1.190173
Adjusted R-squared		S.D. dependent var	0.723198
S.E. of regression	0.682383	Akaike info criterion	2.080533
Sum squared resid	197.8997	Schwarz criterion	2.108985
Log likelihood	-442.2341	Hannan-Quinn criter.	2.091770
F-statistic	27.30365	Durbin-Watson stat	1.972603
Prob(F-statistic)	0.000000		

(a) Calculate the values for the Adjusted R-squared and the t-statistic for the estimated coefficient of LOG(WA).

[5 marks]

(b) Test the hypothesis that LOG(WA) does not affect LOG(WW). Carefully state what alternative hypothesis and significance level you are using. Are your results sensitive to the choice of significance level?

[5 marks]

(c) Construct a 95% confidence interval (using a critical value of 1.96) for the coefficient of LOG(WE), and carefully interpret its meaning.

[5 marks]