Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
Midterm 2, Nov. 04, 2014

Instructor: **Ryan Godwin**
Instructions: **Answer ALL QUESTIONS, and put all answers in the booklet provided**
Time Allowed: **75 minutes (Total marks = 73 – *i.e.,* ~ *one mark per minute*)**
Number of Pages: **3**

PART A: *State whether each of the following is TRUE or FALSE, and BRIEFLY explain your answer. Each question is worth 5 marks. Of these, 4 marks are given for the explanation.*

1.) If we apply a Hausman test of the hypothesis that the errors in a regression model are asymptotically uncorrelated with the regressors, we would use I.V. estimation if the p-value for the test is large enough (say, greater than 5% or 10%).

2.) The Wald test is better than the F-test.

3.) If we fit a linear regression model using Instrumental Variables estimation, with an equal number of regressors and instruments, the residuals will sum to zero as long as the instrument matrix includes a column of 'ones'.

4.) There is no way to compare the asymptotic efficiency of two mean-square consistent estimators, since in both cases the asymptotic distribution degenerates to a "spike".

5.) OLS will be inconsistent if a relevant variable (one which affects $\boldsymbol{y}$), is unobservable. In this case, instrumental variables may be used, as long as the instrument is correlated with the unobservable variable.

PART B: Answer 3 out of 4 questions. **Only** the first 3 questions will be marked.

6.) Consider the population model:

$$\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\boldsymbol{b}$ is the OLS estimator for $\boldsymbol{\beta}$.

a) Prove that the $plim(\boldsymbol{b}) = \boldsymbol{\beta}$, stating any assumptions that you use.

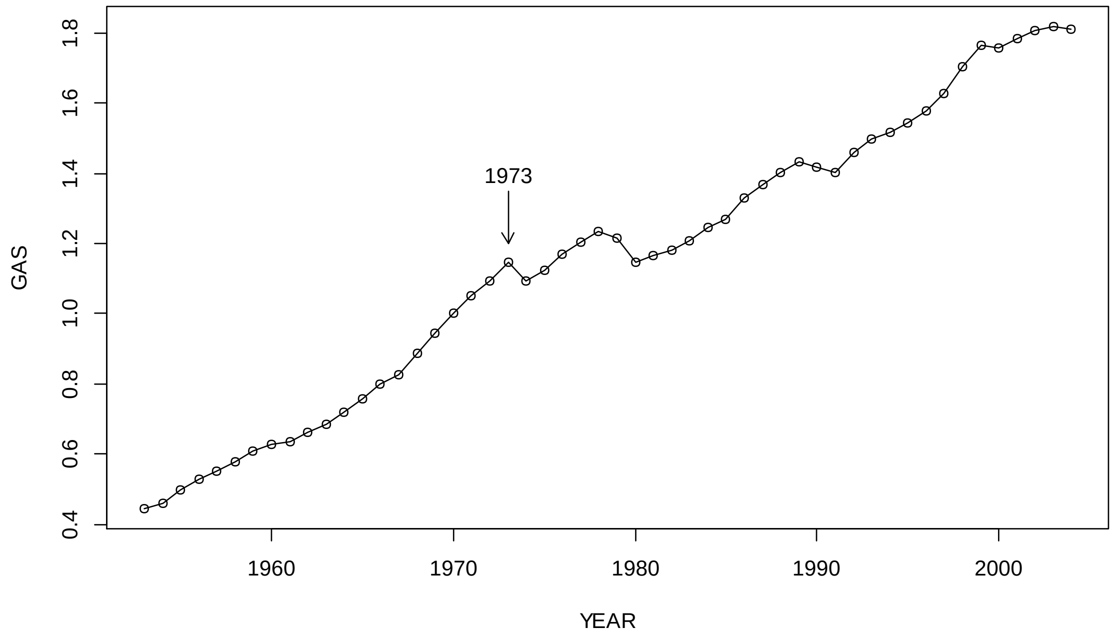
[10 marks]

b) Briefly discuss the costs of including an irrelevant regressor, and excluding a relevant one.

[6 marks]

7.) The following question considers the possibility of a structural break in gasoline consumption:

Per Capita Gasoline Consumption (U.S.A.)



Variables in the data include:

- *GAS* – per-capita gasoline consumption
- *GASP* - the price of gasoline
- *PNC* - the price of new cars
- *PUC* - the price of used cars
- *Income/Pop* – per-capita income

a) Suppose that you want to test whether or not the effect of *GASP* on *GAS* is different for *pre*-1973 and *post*-1973. Suppose also that you strongly believe that all other effects are the same across 1973. What regression model might you specify? What is your null hypothesis, written in terms of the parameters ($\boldsymbol{\beta}$)?

[10 marks]

b) How does the model and null hypothesis in part (a) compare to the Chow test?

[6 marks]

8.) The formula for the F-statistic is:

$$F = \frac{(R\boldsymbol{b} - \boldsymbol{q})'[R(X'X)^{-1}R']^{-1}(R\boldsymbol{b} - \boldsymbol{q})}{\sigma^2}\left(\frac{1}{J}\right)\left(\frac{\sigma^2}{s^2}\right).$$

The residual vector from Restricted-Least-Squares (RLS) estimation may be written as:

$$\boldsymbol{e}_* = \boldsymbol{e} + X(X'X)R'[R(X'X)^{-1}R']^{-1}(R\boldsymbol{b} - \boldsymbol{q}).$$

a) In the case that the model *includes an intercept*, prove that:

$$F = \frac{(\boldsymbol{e}_*{}'\boldsymbol{e}_* - \boldsymbol{e}'\boldsymbol{e})/J}{\boldsymbol{e}'\boldsymbol{e}/(n-k)}.$$

[10 marks]

b) Show that the F-statistic may be calculated using the R^2 from the restricted and unrestricted model.

[6 marks]

9.) Consider the linear multiple regression model, $\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where all of the usual assumptions are satisfied, except that the regressors are random and correlated with the errors (even asymptotically). That is,

$$\text{p}lim\left(\frac{X'\varepsilon}{n}\right) = \gamma \neq 0,$$

and γ is finite. We have available a set of *k* instrumental variables which form the columns of the $(n \times k)$ matrix, *Z*. The *X* and *Z* matrices satisfy the following conditions:

$$plim(X'X/n) = Q_{XX} \quad ; \quad \text{positive definite and finite.}$$
$$plim(Z'X/n) = Q_{ZX} \quad ; \quad \text{positive definite and finite.}$$
$$plim(Z'\varepsilon/n) = 0.$$

a) Discuss the properties of the following estimator of $\boldsymbol{\beta}$:

$$\widehat{\boldsymbol{\beta}} = [A + Z'X]^{-1}Z'y,$$

where *A* is a non-random positive-definite symmetric matrix.

[10 marks]

b) If the matrix *A* were allowed to be random, what further condition(s) would need to be satisfied in order for $\widehat{\boldsymbol{\beta}}$ to still be a "good" estimator?

[6 marks]

END.