The Interdependence of Individual Portfolio Decisions and the Demand for Insurance

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We analyze the individual’s demand for insurance as a special case of general portfolio hedging activity. The demand for insurance contracts is determined simultaneously with the demands for other assets in the portfolio. We demonstrate that when the payoffs of the policy are correlated with the payoffs to the individual's other assets, the demand for insurance contracts is generally not a separable portfolio decision. We argue that this separability condition is not generally met because of significant interdependence of claims across different insurance policies. Furthermore, our generalizations can reverse the standard prediction that wealthier individuals will demand less insurance.

I. Introduction

Most analyses of the demand for insurance assume (1) that there is only one source of uncertainty in the individual's opportunity set and (2) that an insurance contract is the only available asset for hedging risk (see, e.g., Arrow 1963; Mossin 1968; Pauly 1968; Smith 1968; Gould 1969; Zeckhauser 1970; Ehrlich and Becker 1972; Razin 1976; Harris and Raviv 1978; Spence 1978; Holmström 1979; Shavell 1979). But insurance contracts are only a subset of the assets of an

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individual's portfolio. As Gould (1969, p. 151) notes in discussing potential generalizations of his analysis: “One important such generalization would be a model that considered all aspects of the consumer's portfolio decision instead of treating his collision insurance choice independently.” Our analysis provides that generalization. We demonstrate that the demand for a specific insurance policy is generally not a separable portfolio decision (see Leontief 1947), that there are potentially important interdependencies among the demands for insurance policies and other assets.

II. A Characterization of the Individual’s Demand for Insurance

In the perfect capital markets framework (e.g., Fama and Miller 1972) there is no demand for specific insurance contracts even though risk aversion and uncertain future consumption opportunities are explicitly assumed. Individuals can sell shares in any claim (including human capital claims) and effectively eliminate insurable risks through diversification. Thus, a necessary condition for a specific demand for insurance is that costs of eliminating risks through diversification exceed the costs of hedging them with insurance (see Benston and Smith 1976; Mayers and Smith 1982). Mayers (1972, 1973) has modified the traditional analysis by assuming that assets are either perfectly marketable (zero contracting costs) or completely nonmarketable such as human capital (infinite contracting costs). This framework is consistent with a specific demand for insurance because it assumes risk-averse individuals, uncertain consumption opportunities, and relevant contracting costs.

We assume that both nonmarketable and perfectly marketable assets exist and that there are two classes of insurable events, which for convenience we call health and liability events. Thus we decompose the individual’s end-of-period random dollar return on his nonmarketable assets into the gross return, \( \hat{Y}_i \); losses generated by liability events, \( \hat{L}_i \); and losses generated by health events, \( \hat{h}_i \). His net nonmarketable asset return is \( \hat{N}_i = \hat{Y}_i - \hat{L}_i - \hat{h}_i \).

Insurance policy payouts are assumed perfectly negatively correlated with the losses from each class of events. Let \( \alpha_i \) and \( \eta_i \) be the individual’s insurance choice variables such that \( \alpha_i \hat{L}_i \) and \( \eta_i \hat{h}_i \) are the payouts actually received from the respective policies. Thus \( \alpha_i \) or \( \eta_i \) equal to one indicates a policy providing full coverage for that loss,

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1 Separability requires that the marginal rate of substitution between any pair of other assets be unaffected by the utilization of this insurance contract.
2 Two events are sufficient to illustrate our point; generalizing to more events is straightforward.
while $\alpha_i$ and $\eta_i$ less than one indicates a policy providing fractional coverage.

To analyze the individual's demand for insurance, we define end-of-period wealth as

$$\bar{R}_i = X^' \bar{R} + N_i + \alpha_i \bar{l}_i + \eta_i \bar{h}_i - r d_i, \quad (1)$$

where $X_i$ is a column vector $(X_i1, X_i2, \ldots, X_iN)'$, $X_j$ is the fraction of firm $j$'s shares held by individual $i$, $\bar{R}$ is a column vector $(\bar{R}_1, \bar{R}_2, \ldots, \bar{R}_N)'$, $\bar{R}_i$ is the end-of-period total dollar value of firm $j$'s shares, $r$ is one plus the one-period riskless rate of return, and $d_i$ is the net debt of individual $i$. We also assume that the individual's preferences are a positive function of expected end-of-period wealth $\bar{R}_i$ and a negative function of the variance of end-of-period wealth $\sigma^2(\bar{R}_i)$,

$$U_i' = U_i'[\bar{R}_i, \sigma^2(\bar{R}_i)], \quad (2)$$

where

$$\frac{\partial U_i'}{\partial \bar{R}_i} \equiv U_i' > 0 \text{ and } \frac{\partial U_i'}{\partial \sigma^2(\bar{R}_i)} \equiv U_i'' < 0.$$

The individual's portfolio/insurance problem is to choose $X_i$, $\alpha_i$, $\eta_i$, and $d_i$ to maximize his preference function (eq. [2]), subject to the budget constraint

$$W_i = X^' P + \alpha_i P_{hi} + \eta_i P_{hi} - d_i, \quad (3)$$

where $P$ is a column vector $(P_1, P_2, \ldots, P_N)'$, $P_j$ is the current total market value of firm $j$'s shares, and $P_{hi}$ and $P_{hi}$ are the premiums for full coverage under the health and liability policies. The solution to this portfolio problem provides a demand equation for each type of insurance policy as well as the individual's demand functions for risky marketable assets.\(^1\)

The demand equations for insurance are symmetric so we need only examine one. The optimal coverage under the liability insurance policy can be characterized as follows:\(^2\)

\(^3\) Thus, in our model the individual's end-of-period wealth is the sum of (1) the end-of-period equity values of his fractional ownerships of the firms in the market, (2) his net nonmarketable asset return, (3) the payoffs from his insurance policies, and (4) the repayment of any riskless loans.

\(^4\) Additional assumptions are infinite divisibility, price-taking behavior, and no short sales constraints. We suppress the risky marketable asset demand functions in our analysis.

\(^5\) This equation is really a characterization since we have not solved out the other choice variables. This form is more interpretable. For example, by defining the liability policy as asset $N + 1$ and the health policy as asset $N + 2$, the full solution, equivalent to eq. (4), is

$$\alpha_i = 1 + k_i \sum_{j=1}^{N+2} Q_{N+1,j}(\bar{R}_i - r P_j) + \sum_{j=1}^{N+2} Q_{N+1,j} \sigma_{ij}.$$
\[
\alpha_i^* = \frac{1}{\sigma^2(\hat{\ell}_i)} \left[ -X_i' \gamma_i - \text{cov} (\hat{l}_i, \hat{Y}_i) + (1 - \eta_i) \text{cov} (\hat{l}_i, \bar{h}_i) \right] \\
+ \frac{1}{\sigma^2(\hat{\ell}_i)} \left[ \sigma^2(\hat{\ell}_i) + k_i (\hat{\ell}_i - rP_{li}) \right],
\]

where \( \gamma_i \) is the column vector \((\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iN})'\), \( \sigma_{ij} \) is the covariance between \( \hat{l}_i \) and \( \hat{R}_j \), \( \text{cov} \) is the covariance operator, and \( k_i = -(U'_{v}/2U_{v}) > 0 \), given \( U'_{v} > 0, U_{v} < 0 \). Thus, \( k_i \) can be interpreted as a marginal rate of substitution between expected return and variance evaluated at the optimum.

The first bracketed term in equation (4) collects the factors that are added by considering the demand for insurance in a portfolio context. These factors all represent substitution alternatives for the insurance policy under consideration. The second bracketed term in equation (4) reflects those factors which would appear in an analysis done in isolation. If the insurance were priced at its actuarial/discounted-at-the-riskless-rate value (i.e., \( P_{li} = \hat{l}_i/r \)) and if the first term were zero, then the individual would demand full coverage: \( \alpha_i^* = [\sigma^2(\hat{\ell}_i)/\sigma^2(\hat{\ell}_i)] = 1 \). If the premium included a loading fee (i.e., \( P_{li} > \hat{l}_i/r \)), the individual would demand less than full coverage. Moreover, the coverage demanded is affected by the consumer’s degree of risk aversion, \( k_i \) (e.g., if wealthier individuals are less risk averse, with positive loading fees, they will have a lower demand for insurance).

III. Separability of the Demand for Insurance

A sufficient condition for the separability of the demand for insurance from other portfolio decisions is that the losses of a particular type are orthogonal to the payoffs to all marketable assets, the individual’s gross human capital, and the losses associated with other insurable events. We examine each term in the first bracketed expression in (4) to see how restrictive this assumption might be.

The first term, \(-X_i' \gamma_i\), could be described as “homemade” insurance. It represents the insurance protection provided by the marketable assets held in the consumer’s optimal portfolio, that is, \(-X_i' \gamma_i = -\text{cov} (\hat{l}_i, \hat{R}_{pi})\), where \( R_{pi} \) is the individual’s marketable asset portfolio return. Of course, how much protection the consumer obtains through homemade insurance depends both on the availability of marketable assets that can provide such a hedge and on the relative costs of a standard insurance policy versus homemade insurance (see Mayers and Smith 1981).\(^6\) Hence risks which have a large market

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\(^6\) One dimension in which the cost should be reduced with homemade insurance is

where \( \sigma_{ij} = \text{cov} (\hat{Y}_i, \hat{R}_j), j = 1, \ldots, N + 2, \) and \( Q_{N+1} \) are elements from row \( N + 1 \) of the inverse of the \((N + 2) \times (N + 2)\) variance-covariance matrix.
component are likely to be hedged through homemade insurance. Moreover, since the dollar magnitude of the individual’s risky marketable asset holdings enters in the term \(X_i'\gamma\), for two individuals who face the same risk \(\bar{I}_i\) and hold the same proportions of marketable securities, but one with twice the marketable assets as the other, assuming \(\text{cov}\ (\bar{I}_i, \bar{R_p}) > 0\), the individual with the greater marketable wealth will demand less insurance. Although there is some evidence that suggests there is a market component to health-related events, we doubt that the magnitude of the effect is empirically important in determining the demand for health insurance.\(^7\)

The second term, \(-\text{cov}\ (\bar{I}_i, \bar{Y}_i)\), represents the individual’s incentive to self-insure.\(^8\) If liability losses are likely to be large in states of the world that are otherwise “bad,” the consumer will demand more insurance. For example, a physician who suffers a successful malpractice suit would expect the demand for his services to fall. This would make the \(\text{cov}\ (\bar{I}_i, \bar{Y}_i)\) negative and his demand for liability insurance is correspondingly higher. Similarly, if claims under health insurance policies tend to be associated with reductions in productivity, the demand for health insurance is higher. Nonmarketable wealth also enters directly into the \(\text{cov}\ (\bar{I}_i, \bar{Y}_i)\) term. If one consumer has twice the return \(\bar{Y}_i\) as another in every state of nature and if they both face the same distribution of liability losses, the wealthier individual will tend to purchase more insurance assuming \(\text{cov}\ (\bar{I}_i, \bar{Y}_i) < 0\). This will tend to reverse the usual implication that individuals with greater wealth will demand less insurance.

The final term, \((1 - \eta_i)\text{cov}\ (\bar{I}_i, \bar{h}_i)\), reflects the possibility of substitutions between insurance of different types where the risks are not orthogonal. We think that quantitatively the dependence of payoffs across different insurance policies is the most important of the factors isolated in our analysis. There are likely to be important correlations in claims across different insurance policies; for example, given a claim under an automobile insurance policy, the probability of a claim under health, life, or liability policies is likely to be increased. Thus

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\(^7\) Gerald Seib reports: “An expert calculates that over a six-year period, a one-percentage-point rise in unemployment causes about 37,000 deaths. ‘Virtually all major illnesses, virtually all major causes of death are affected,’ he says” (Wall Street Journal [August 25, 1980], p. 17, “Recessions Cause Death Rates to Rise as Pressures of Coping Take Hold”).

\(^8\) This incentive to self-insure does not derive from the insured’s “willingness” to bear risk. Quite the contrary, it derives from his aversion to risk; he retains some of the risk for diversification purposes. Consider the problem of choosing \(\alpha\), to minimize the variance of \(\bar{R}_i \equiv \bar{Y}_i - \bar{I}_i + \alpha \bar{h}_i\). The solution is \(\alpha_i = 1 - \text{cov}\ (\bar{I}_i, \bar{Y}_i)/\sigma^2(\bar{I}_i)\), not \(\alpha_i = 1\).
with more than one type of risk, where the risks are correlated, and specialization exists across insurance policies there is an indeterminacy with regard to the demand for a particular insurance contract.\footnote{The interdependencies in demand across insurance contracts could be internalized if insurance coverage were provided through a single blanket insurance policy. Observed contracts typically cover a collection of related hazards; we believe that the analysis of the covariance in payoffs across hazards will be important in explaining how coverage is bundled across contracts.}

IV. Conclusions

Much of the analysis of the demand for insurance focuses on insurance in isolation. Yet insurance is a special case of hedging risk by risk-averse individuals. Capital markets provide a rich array of contractual forms convenient for risk reduction; insurance contracts are but a subset of this array of available alternatives. We examine the interrelationship between insurance holdings and other portfolio decisions. Sufficient conditions for insurance decisions to be independent of other portfolio decisions are: (1) There is no moral hazard or adverse selection; and (2) the payoffs to the insurance policy are orthogonal to those of all marketable securities, the consumer’s gross human capital, and the payoffs to other insurance policies. Although the first restriction is well known, the second has been unrecognized. Moreover, we argue that this omission is not trivial. There are potentially important covariances in the payoffs with other insurance policies and with human capital which lead to different predictions about insurance demands than obtained under the assumption of separability.

Our analysis has abstracted from a number of important considerations which also affect the structure of observed insurance contracts. For example, we have ignored the general prohibition against insurance exceeding 100 percent of value. This prohibition obviously constrains the ability of individuals to purchase more than full coverage of one type of insurance as a partial hedge against another risk. However, we have also omitted consideration of moral hazard and loading fees which lead to an optimal choice of less than full coverage when demands are separable (see Mayers and Smith 1981). In general, incorporation of these considerations should not overturn the qualitative implications of the interdependencies on which we focus.

Some types of analyses will be more robust than others in ignoring portfolio considerations. For example, analyses concerned with the impact of administrative expenses on the structure of the deductible schedule of an insurance contract are likely to be little affected. On the other hand, analyses of the type done by Pashigian, Schkade, and

\footnote{The interdependencies in demand across insurance contracts could be internalized if insurance coverage were provided through a single blanket insurance policy. Observed contracts typically cover a collection of related hazards; we believe that the analysis of the covariance in payoffs across hazards will be important in explaining how coverage is bundled across contracts.}
Menefee (1966), Gould (1969), and Drèze (1981), where the question concerns whether observed individual demand behavior conforms to the predictions of a model that does not consider alternatives for risk reduction, are less likely to be robust (a point admitted by Gould).

References


Shavell, Steven. “Risk Sharing and Incentives in the Principal and Agent Relationship.” Bell J. Econ. 10 (Spring 1979): 55–73.
