Inaugural Annual Meeting of PNRMS

The inaugural annual meeting of the PNRMS was held at University of Regina, May 3–4, 2007. The Organizing Committee of the inaugural meeting consisted of a member from each of the participating universities as follows: Murray Bremner (University of Saskatchewan), Douglas Farenick, Chair (University of Regina), Chenkuan Li (Brandon University), Anna Stokke (University of Winnipeg), and Nina Zorboska (University of Manitoba). The meeting, which attracted over 40 researchers from 5 universities within the provinces of Manitoba and Saskatchewan, featured 14 research talks in diverse areas of mathematics (each of 50 minutes duration) over the 2-day period. Posters were also presented. A noticeable feature of the Inaugural Meeting was the participation of numerous younger researchers and women. The Program Schedule of the meeting, together with abstracts of talks and posters and list of participants of the meeting are given below.

Scientific Program

May 3, 2007

08.30 - 09.20. Shaun Fallat, Applications of total positivity

09.30 - 10.20. Stephanie Portet, Mathematical modeling of the cytoskeleton

10.20 - 10.40. Refreshment Break

10.40 - 11.30. Michael Kozdron, A random look at the Schramm-Loewner evolution

11.40 - 12.30. Eric Shippers, Quasiconformal Teichmueller theory

12.30 - 14.10. Lunch

14.10 - 15.00. Anna Stokke, Quantum Schur algebras and Desarmenien matrices

15.10 - 16.00. Salma Kuhlmann, Approximation of positive polynomials by sums of squares

16.00 - 16.20. Refreshment Break

16.20 - 17.10. Morten Nielsen, A cycle partition problem for graphs

17.20 - 18.10. Chenkuan Li, A review of products of distributions

18.30 - 22.00. Conference Banquet (University Club)

May 4, 2007

08.30 - 09.20. Murray Bremner, Structure constants for the enveloping algebra of the five-dimensional non-Lie Malcev algebra

09.30 - 10.20. Michael Roddy, Algorithmic approaches to the Product Problem for the infinite case

10.20 - 10.40. Refreshment Break

10.40 - 11.30. Vaclav Linek, New results on polyhedral designs

11.40 - 12.30. Jaydeep Chipalkatti, On equations defining coincident root loci

12.30 - 14.10. Lunch

14.10 - 15.00. Mikhail Kotchetov, Group gradings on simple Lie algebras

15.10 - 16.00. Remus Floricel, E_0-semigroups on von Neumann algebras

16.00 - 16.10. Adjournment

Abstracts

- Shaun Fallat, University of Regina, *Applications of total positivity.* An $m \times n$ real matrix is called totally positive (resp. totally nonnegative) if all of its minors are positive (resp. nonnegative). This class of matrices grew out of three separate applications: Vibrating systems, interpolation, and statistics. Since the pioneering work of Gantmacher/Krein, Schoenberg, and Karlin, the class of totally positive matrices has evolved into an important subject area in mathematics and it continues to arise in numerous applications including: Weighted planar networks (ballot numbers); computer aided geometric design (shaper preserving transformations); probability (moment matrices); and geometry (McMullen correspondence). I intend to survey a number of current applications involving this class (including some mentioned above), and if time permits I will highlight some recent accomplishments connecting the eigenvalues of totally nonnegative matrices to the roots of certain biorthogonal polynomials.

- Stephanie Portet, University of Manitoba, *Mathematical modeling of the cytoskeleton.* The cytoskeleton is a complex arrangement of structural proteins organized in networks: microfilaments, intermediate filaments and microtubules. Each network has specific properties and organization as well as particular roles in the cell. The organization of a cytoskeletal network is the main determinant of its cellular function. I will present some mathematical models of cytoskeletal networks.

- Michael Kozdron, University of Regina, *A random look at the Schramm–Loewner evolution.* The stochastic Loewner evolution (SLE) is a one-parameter family of random geometric processes in the complex plane introduced by Oded Schramm in 1999 which is believed to describe the scaling limit of a variety of statistical mechanics models. Recently a number of rigorous results about such scaling limits have been established; in fact, Wendelin Werner was awarded the Fields Medal in 2006 for "his contributions to the development of stochastic Loewner evolution, the geometry of two-dimensional Brownian motion, and conformal field theory." In this talk I will introduce SLE, describe some of its basic properties including the relationship between SLE, Brownian motion, and CFT, and illustrate its connection to a variety of models including the Ising model, self-avoiding walk, loop-erased random walk, and percolation.

- Eric Shippers, University of Manitoba, *Quasiconformal Teichmuller theory.* Quasiconformal Teichmueller theory is the study of deformations of complex structures of Riemann surfaces. It plays an important role in many fields, such as complex dynamics, hyperbolic geometry and conformal field theory. This talk is a non-technical introduction to the main ideas of Teichmueller theory and quasiconformal maps. If time allows recent applications to conformal field theory will be discussed.

- Anna Stokke, University of Winnipeg, *Quantum Schur algebras and Desarmenien matrices.* The classical Schur algebras play an important role in the representation theory of the general linear groups. We will discuss quantized versions of these algebras, the quantum Schur algebras, and discuss how they can be used to define upper

triangular matrices called the quantum Désarménien matrices. Among other things, the quantum Désarménien matrices can be used to give "straightening algorithms" for quantum bideterminants.

- Salma Kuhlmann, University of Saskatchewan, *Approximation of polynomials by sums of squares.* Approximation of positive polynomials by sums of squares has important applications to polynomial optimisation. In this talk, I will survey the main recent results achieved on that topic: I will consider positive (respectively, non-negative) polynomials on compact (respectively, unbounded) semi-algebraic sets. I will discuss representations in the associated preorderings (respectively, linear representations in the associated quadratic module). The representation often depends on the dimension of the semi-algebraic set; I will present stronger results in the low dimensional case. I will also highlight special representations when the positive polynomials under consideration are sparse (that is, satisfy some separation and overlap conditions on the variables appearing in the monomials).

- Morten Nielsen, University of Winnipeg, *A cycle partition problem for graphs.* A graph is a pair $G = (V, E)$ consisting of a finite set V of vertices and a set $E \subseteq V \times V$ of unordered pairs of vertices, called edges. The number $|V|$ is called the order of G. A cycle of order (or length) k in G is a sequence $C = v_0 v_1 \dots v_{k-1} v_0$ of k distinct vertices in$G$ such that $v_i v_{i+1} \in E$ for $i \geq 0$. The circumference of G, denoted $c(G)$, is the order of the longest cycle in G. For a subset $S \subseteq V$ we denote by $\langle S \rangle$ the subgraph induced by the vertices of S. We consider the following problem: which graphs G have the property that for every choice of two positive integers c_1 and c_2 with sum $c(G)$, there exists a partition $V_1 \cup V_2 = V$ of the vertex set of G such that $c(\langle V_i \rangle) \leq c_i$, for $i = 1, 2$.

- Chenkuan Li, Brandon University, *A review of products of distributions.* The problem of defining products of distributions has been an open and active research area since Schwartz introduced the theory of distributions around 1950. The inherent difficulties of obtaining products have never prevented their appearance in literature, as they are needed in quantum field theory and differential equations with distributions involved. In this talk, we recollect various approaches, which include sequential and complex analysis methods, to tackling products of distributions in one or multiple variables, as well as particular generalized functions defined on certain manifolds.

- Murray Bremner, University of Saskatchewan, *Structure constants for the universal enveloping algebra of the five-dimensional nilpotent non-Lie Malcev algebra.* We determine explicit structure constants for the nonassociative multiplication in the universal enveloping algebra $U(M)$ of the 5-dimensional nilpotent non-Lie Malcev algebra M by constructing a representation of $U(M)$ in terms of differential operators on the polynomial algebra $P(M)$. As a corollary we show that $U(M)$ is not power-associative. Our starting point is the recent theorem of Pérez-Izquierdo and Shestakov which extends to Malcev algebras the Poincaré–Birkhoff–Witt theorem for Lie algebras.

- Michael Roddy, Brandon University, *Algorithmic approaches to the Product Problem for the infinite case.* The Product Problem for the Fixed Point Property in ordered

sets is still open if both factors are infinite. In 2003 Bernd Schroeder and I proposed a method for attacking the problem in the infinite case by revisiting Walker's old paper on Isotone relations. Walker's condition is equivalent to dismantlability. Our idea was to weaken his conditions in the direction of FPP while maintaining the proof of the Product Problem he provided. Verifying these weaker properties in small ordered sets with FPP turned out to be a very difficult process. I had a group of students working on this in the summer of 2003. The purpose of this talk will be to revisit this work and to receive feedback on the feasability of perhaps continuing this project.

- Vaclav Linek, University of Winnipeg, *New results on polyhedral designs.* A Steiner Quadruple System of order v, or SQS(v), is a pair (X, B) where $|X| = v$ and B is a collection of 4-subsets of X such that any 3-subset of X is in a unique quadruple of B. If we view 4-subsets of B as unoriented tetrahedra whose vertices are labelled with elements of X, then each unoriented triangle labelled with distinct elements of X occurs exactly once as a 3-face of some tetrahedron in B. So one can call a SQS(v) an "unoriented tetrahedral design," and a host of questions arises about the analogous (oriented or unoriented) octahedral, cube, icosahedral and dodecahedral designs. We survey the work of Hanani, Hartman and Phelps in this area, and give many new results on these polyhedral designs (joint work with Brett Stevens).

- Jaydeep Chipalkatti, University of Manitoba, *On equations defining coincident root loci.* It is classically known that a polynomial equation

$$\mathbf{F} = a_0\, x^d + a_1\, x^{d-1} + \cdots + a_d = 0, \quad a_i \in \mathbf{C}$$

  has a repeated root, if and only if the discriminant $\Delta_F(a_0, \ldots, a_d)$ is zero. Can we find similar algebraic conditions for $\mathbf{F}$ to have (say) a triple root or two pairs of repeated roots? We will survey the many versions in which this question can be asked, and the sort of answers one may expect. Along the way we will introduce some ideas from classical invariant theory and the representation theory of SL_2. I will explain a recent result due to myself and A. Abdesselam (Paris XIII), where we can write down such conditions using $3j$-symbols from the quantum theory of angular momentum.

- Mikhail Kotchetov, Memorial University of Newfoundland, *Group gradings on simple Lie algebras.* Group gradings on algebras, especially simple algebras, have been extensively studied since the 1960s. In particular, gradings on Lie algebras arise in the theory of symmetric spaces, Kac-Moody algebras, and Lie coloralgebras. In the context of simple Lie algebras, it suffices to consider only gradings by abelian groups (since the support of any grading generates an abelian group). In 1968, V. Kac classified all gradings by cyclic groups on finite-dimensional simple Lie algebras. We will discuss recent progress in the classification of gradings on finite-dimensional simple Lie algebras by arbitrary abelian groups.

- Remus Floricel, University of Regina, *E_0-semigroups on von Neumann algebras.* This talk will introduce the theory and current problems of interest for semigroups of endomorphisms of von Neumann algebras.

Posters

- Mahshid Atapour, University of Saskatchewan, *Entanglement complexity of polymer systems in tube.* In this poster presentation we give some rigorous results on a model of polymers in dense systems such as melts. Our work is a first step in building a theoretical framework for understanding the Orlandini et al 2004 model. We imagine cutting an infinite rectangular tube out of the system. This tube will capture several polymers running through its interior, and starting and ending on the boundary. These subchains will be mutually entangled. Associating a 2-component link to each pair of polymers, we use the linking number to measure the entanglement complexity of polymer systems. We model these polymer systems by systems of self-avoiding walks (SSAWs), then we prove a pattern theorem for SSAWs. As a result, we rigorously prove that there exists a positive number b such that the probability that a polymer system of size n has entanglement complexity greater than bn approaches 1 as $n \to \infty$. Furthermore, in a special case we prove that the linking number grows exactly linearly in n.

- Michael Cavers, University of Regina, *Inertially arbitrary sign pattern matrices.* In economics and other disciplines, it is important to be able to solve certain problems based only on the signs of the entries of the matrices involved. This gives rise to sign patterns, that is, a matrix whose entries are from the set $\{+, -, 0\}$. Information about the eigenvalues of a matrix can often be obtained by looking at its sign pattern. Certain properties that the eigenvalues of such a matrix must satisfy are looked at, as well as the effect these conditions impose on its sign pattern. In particular, we focus on the property that a sign pattern is inertially arbitrary, that is, allows any inertia.

- Ryan Henry, Brandon University, *A class of irreducible union-closed families.* This poster presentation will give a brief introduction to irreducible union-closed families of sets. We will show how this concept may provide insight into the union-closed sets conjecture, and summarize the results of exploratory research done on one such class of irreducible families.

List of Participants

1. Elliott, Carrie; Brandon University (undergrad)
2. Henry, Ryan Brandon University (undergrad)
3. Li, Chenkuan; Brandon University (faculty)
4. Roddy, Michael; Brandon University (faculty)
5. Tang, Ricky; Brandon University (undergrad)
6. Kotchetov, Mikhail; Memorial U. (faculty)
7. Chipalkatti, Jaydeep; University of Manitoba (faculty)
8. Gumel, Abba; University of Manitoba (faculty)
9. Moghaddam, Hossein; University of Manitoba (faculty)
10. Mohammed, Tagreed; University of Manitoba (grad)
11. Portet, Stephanie; University of Manitoba (faculty)
12. Schippers, Eric; University of Manitoba (faculty)
13. Argerami, Martin; University of Regina (faculty)
14. Cavers, Michael; University of Regina (grad)
15. Dean, Matthew; University of Regina (pdf)
16. Fallat, Shaun; University of Regina (faculty)
17. Farenick, Doug; University of Regina (faculty)
18. Fisher, Chris; University of Regina (faculty)
19. Floricel, Remus; University of Regina (faculty)
20. Guo, Chun-Hua; University of Regina (faculty)
21. Kirkland, Steve; University of Regina (faculty)
22. Kozdron, Michael; University of Regina (faculty)
23. Liu, Xiaoping; University of Regina (grad)
24. Massey, Pedro; University of Regina (pdf)
25. Mitra, Dipra; University of Regina (grad)
26. Stanley, Donald; University of Regina (faculty)

27. Szechtman, Fernando; University of Regina (faculty)
28. Tifenbach, Ryan; University of Regina (grad)
29. Volodin, Andrei; University of Regina (faculty)
30. Xing, Yongyun; University of Regina (grad)
31. Anvari, M.H.; University of Saskatchewan (faculty)
32. Atapour, Mashid; University of Saskatchewan (grad)
33. Bremner, Murray; University of Saskatchewan (faculty)
34. Douglas, Andrew; University of Saskatchewan (pdf)
35. Kuhlmann, Franz-Viktor; University of Saskatchewan (faculty)
36. Kuhlmann, Salma; University of Saskatchewan (faculty)
37. Marshall, Murray; University of Saskatchewan (faculty)
38. Pereira, Rajesh; University of Saskatchewan (faculty)
39. Linek, Vclav; University of Winnipeg (faculty)
40. Nielsen, Morten; University of Winnipeg (pdf)
41. Stokke, Anna University of Winnipeg (faculty)

Signed (PNRMS Coordinating Committee)

Professor Abba Gumel (Manitoba)

Professor Stephen Kirkland (Regina)

Professor Ray Spiteri (Saskatchewan)