Fast and Accurate Analysis of Thin Shields with Holes Based on the Current Sheet Integral Equation

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Abstract — The distribution of induced electric current in thin metallic shields with holes is determined by using the current sheet integral equation. A polyhedral surface with triangular elements is chosen to model the shield surface. The surface density of current is taken to be uniform over each element and is described by using specialized vector functions whose coefficients are scalar quantities associated with the interior nodes and with the holes in the shield. A Galerkin method is used to solve the integral equation. Numerical experiments show that the proposed method is more efficient than other methods applied so far to thin shields with holes. Once the current distribution is computed, the magnetic induction, the shielding efficiency and the forces exerted on the shield can rapidly be evaluated.

Index Terms — Current sheet integral equation, magnetic forces, shielding efficiency.

I. INTRODUCTION

A practically sufficient shielding efficiency could be achieved by employing thin metallic shields with holes, which reduces their cost and weight, the holes also allowing for an adequate installation of the measurement and control devices. Whenever the shield thickness is much smaller than the depth of penetration of the electromagnetic field, the induced current can be considered to be distributed in the form of a current sheet over its surface. The magnetic field can be determined by introducing a surface impedance [1]-[3], but this requires a filed problem solution over the entire region where the shield is located. A Lagrangian equation formulation has been presented in [4] for shields without holes, with the current density being expressed in terms of a scalar function such that the current continuity conditions are fulfilled.

A novel approach is proposed in [5] for simply connected shields, where the surface current density is represented as a linear combination of specialized vector functions associated with the interior nodes of the polyhedral mesh with triangular surface elements employed to approximate the shield surface. For multiply connected shields a set of vector functions is added, each function being associated with only one hole of the shield, thus reducing the number of unknowns involved [6].

In the present work, one applies the method in [6] for the numerical solution of the current sheet integral equation for a multiply connected thin shield in order to determine its shielding efficiency and also the electromagnetic forces acting on it.

II. SOLUTION OF CURRENT SHEET INTEGRAL EQUATION

The surface current density $J_s$ of the shield sheet satisfies the integral equation

$$\frac{1}{\sigma_s} J_s(r) + \frac{\mu_0 f}{2} \oint_S J_s(r') dS' = -j 2 \pi \mu_0 A_0(r) - \nabla V, \ r \in S$$  \hspace{1cm} (1)

where $\sigma_s = \sigma \Delta$ is the shield surface conductivity, $\Delta$ the shield thickness, $j = \sqrt{-1}$, $f$ the frequency, $\mu_0$ the permeability of free space, $R = |r - r'|$ with $r$ and $r'$, respectively, the position vectors of the observation point and of the source point, $A_0$ is the magnetic vector potential due to external sources, and $-\nabla V$ the scalar potential component of the electric field intensity. $J_s$ over the surface $S$ of the shield is expressed as

$$J_s(r) = \sum_{i=1}^{N} \alpha_i U_i(r) + \sum_{m=1}^{N_h} \beta_m W_m(r)$$  \hspace{1cm} (2)

with the function $U_i$ associated with the interior node $i$ and taken to be $U_i(k) = \frac{1}{2 S_k}$ when the node $i$ belongs to the triangular surface element $k$ of area $S_k$, $l_i(k)$ being the length vector along the edge of the element $k$ that is opposed to the node $i$, as shown in Fig. 1, and with the function $W_m$ associated with each rim $m$ of the $N_h$ holes of the shield, defined as

$$W_m(r) = \sum_{i \in \{m\}} U_i(r)$$  \hspace{1cm} (3)
Fig. 1. A node \( i \) and associated surface elements.

\( \{m\} \) representing the set of nodes \( i \) on the rim of the hole \( m \).

Projecting (2) on the \( N \) functions \( U_n \), \( n = 1,2,\ldots,N \), and on the \( N_h \) functions \( W_n \), \( n = 1,2,\ldots,N_h \), one obtains a system of \( N + N_h \) equations in the unknowns \( \alpha_i \) and \( \beta_m \) in the form [6]

\[
\sum_{i=1}^{N} A_{ni} \alpha_i + \sum_{m=1}^{N_h} B_{nm} \beta_m = C_n, \quad n = 1,2,\ldots,N + N_h.
\]

III. FLUX DENSITY AND SHIELDING EFFICIENCY

The magnetic induction due to the induced currents is

\[
B(r) = \sum_{k=1}^{F} B^{(k)}(r)
\]

where \( F \) is the number of triangular facets of the approximating polyhedral surface and \( B^{(k)} \) is the magnetic induction contribution of the surface element \( k \), i.e.,

\[
B^{(k)}(r) = \frac{\mu_0}{4\pi} J_s^{(k)} \times \int \frac{R}{S_k R^3} dS',
\]

\[
J_s^{(k)} = \frac{\mu_0}{4\pi} \left( \frac{1}{2S_k} \sum_{i \in \{k\}} \gamma_i l_i^{(k)} \right) \times \int \frac{R}{S_k R^3} dS'.
\]

\( J_s^{(k)} \) being the surface current density over the element \( k \), and \( \{k\} \) the set of the nodes \( i \) belonging to the element \( k \), \( \gamma_i \) has the value \( \alpha_i \) for an interior node \( i \) and \( \beta_m \) for the nodes \( i \) belonging to the contour of the hole \( m \). Since (see Fig 1)

\[
l_i^{(k)} \times \frac{R}{R^3} = \left( n_k \frac{l_i^{(k)} \times R}{R^3} \right) n_k + \left( h_i^{(k)} \frac{l_i^{(k)} \times R}{R^3} \right) h_i^{(k)}
\]

where \( n_k \) and \( h_i^{(k)} \) are, respectively, the unit vector normal to and the altitude vector of the node \( i \) for the element \( k \), we have

\[
\int l_i^{(k)} \times \frac{R}{R^3} dS' = -\left( l_i^{(k)} \cdot \frac{dR}{\partial s_k} \right) n_k - l_i^{(k)} \frac{h_i^{(k)} R}{5_k} \int \frac{n_k R}{R^3} dS'.
\]

\[
Y_k(r) \equiv \int \frac{dR}{\partial s_k}
\]

with \( \Omega_k \) the solid angle under which the surface element \( S_k \) is seen from the observation point \( P \). \( Y_k \) and \( \Omega_k \) are evaluated from exact analytical expressions. Finally, the contribution of the surface element \( k \) to the magnetic induction can be written in the form

\[
B^{(k)}(r) = \frac{\mu_0}{8\pi} \left( \left( J_s^{(k)} Y_k(r) \right) n_k - J_s^{(k)} \times n_k \Omega_k(r) \right)
\]

where

\[
J_s^{(k)} = \frac{1}{2S_k} \sum_{i \in \{k\}} \gamma_i l_i^{(k)}.
\]

The shielding efficiency at any point is defined as

\[
\eta = \frac{B_T}{B_0}
\]

where \( B_0 \) is the rms value of the magnetic induction \( B_0 \) due to the external field in the absence of the shield and \( B_T \) that of the magnetic induction \( B_T \) in the presence of the shield, i.e.,

\[
B_T = B_0 + B.
\]

For a global evaluation of the shielding efficiency at \( M \) points chosen within the region of interest, we determine

\[
\eta_g = \frac{\sum_{P=1}^{M} B_{TP}}{\sum_{P=1}^{M} B_{0P}}.
\]

IV. ELECTROMAGNETIC FORCES

Usually, at low frequencies, the electromagnetic shields are utilized in order to lower the field produced by intense currents. Forces exerted on such shields could take big values and their calculation is necessary for an appropriate design of the shields. The average value of the force can be determined by integrating the average value of the normal component of
the Maxwell stress tensor over a closed surface $\Sigma$ enclosing the shield, i.e.,

$$\vec{F} = \frac{1}{\mu_0} \int_0^T \int_\Sigma \left[ \nabla \cdot \left( \mu_0 \nabla \times \vec{B}_T + \mu_0 \nabla \times \vec{B}_0 \right) \right] dS \, dt$$

(15)

where $\vec{B}_T(r,t)$ is the instantaneous value of the magnetic induction. Employing the phasor representation in (13), we have

$$\vec{F} = \frac{1}{\mu_0} \int \int_\Sigma \left[ \text{Re} \left( \nabla \cdot \left( \mu_0 \nabla \times \vec{B}_T \right) \vec{B}_T^* \right) - \mu_0 \frac{\vec{B}_T^2}{2} \right] dS$$

(16)

with * indicating the complex conjugate. Taking into account the superposition in (13) and the fact that only the interaction between the external field and the field due to the induced currents contributes to the resultant force yields

$$\vec{F} = \frac{1}{\mu_0} \int \int_\Sigma \left[ \text{Re} \left( \nabla \cdot (\mu_0 \nabla \times \vec{B}_0) \vec{B}_0^* + \nabla \cdot \vec{B} \vec{B}_0^* \right) - \mu_0 \text{Re} \left( \vec{B}_0 \vec{B}_0^* \right) \right] dS$$

(17)

V. COMPUTATIONAL RESULTS

Consider a copper shield of resistivity $\rho = 2 \times 10^{-8}$ Ohm and thickness $\Delta = 1$ mm occupying the tip of a paraboloid of revolution of equation $x^2 + y^2 = z/5$, with $(x^2 + y^2)^{1/2} \in [0,100]$ mm, which has seven holes as shown in Fig. 2. A coaxial circular turn of a diameter $D = 100$ mm and carrying a sinusoidal current of frequency $f = 5$ kHz and of rms value of 10 kA is located in the $z = 0$ plane. The mesh employed for the current density and magnetic field

Fig. 2. Mesh employed for a paraboloidal thin shield with seven holes.

Fig. 3. Current lines for a phase of $\pi = 58$ degrees of the coil current, frequency $f = 5$ kHz, shield thickness $\Delta = 1$ mm and coil diameter $D$ of: (a) 100 mm, (b) 200 mm.

Fig. 4. The polyhedral surface $\Sigma$ enclosing the shield used for the electromagnetic force calculation.
computations is shown in Fig. 2 and has 2,217 nodes and 3,736 triangular facets, which yields 2,224 unknowns $i$ and $m$. The current lines are sketched in Fig. 3(a). The shielding efficiency is computed for points on the line segment $x \in (-75 \text{ mm}, 75 \text{ mm})$, $y = 0$, $z = 20 \text{ mm}$ and is given in Fig. 5(a). The global shielding efficiency (14) for these points is 0.125. The average electromagnetic force in (17) is calculated by choosing a closed polyhedral surface $\Sigma$ (as shown in Fig. 4) with 2,040 triangular facets, the magnetic induction $B$ at the centers of these facets due to the induced currents being evaluated with (5), (10), while $B_0$, due to the current turn, is obtained by applying the Biot-Savart formula. The electromagnetic force exerted on the shield has the components $F_x = 0.407 \text{ N}$, $F_y = -0.667 \text{ N}$, $F_z = 161.61 \text{ N}$.

For a circular turn of a diameter $D = 200 \text{ mm}$, the field lines and the shielding efficiency at selected points are shown in Figs. 3(b) and 5(b), respectively. The global efficiency for these points is 0.120 and the resultant force has the components $F_x = 0.0367 \text{ N}$, $F_y = -0.244 \text{ N}$, $F_z = 80.75 \text{ N}$.

The global shielding efficiency and the $z$-component of the force for different frequencies $f$ of the external field, thicknesses $\Delta$, and field coil diameters $D$ are given in Table I.

### Table I: Global Shielding Efficiency and Forces for Various Shields and Inducing Coils

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$D$ (mm)</th>
<th>$\Delta$ (mm)</th>
<th>Global Efficiency</th>
<th>Force $F_z$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>2000</td>
<td>1</td>
<td>0.339</td>
<td>0.16</td>
</tr>
<tr>
<td>5000</td>
<td>200</td>
<td>1</td>
<td>0.120</td>
<td>80.75</td>
</tr>
<tr>
<td>5000</td>
<td>100</td>
<td>1</td>
<td>0.125</td>
<td>161.61</td>
</tr>
<tr>
<td>50</td>
<td>2000</td>
<td>5</td>
<td>0.673</td>
<td>0.10</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>5</td>
<td>0.622</td>
<td>49.78</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>5</td>
<td>0.674</td>
<td>64.21</td>
</tr>
</tbody>
</table>

In order to save the necessary construction material and to reduce their weight, the shields are usually designed as metallic plates with holes, without practically affecting the performance of the respective structures with no holes. The method described above for the induced currents and electromagnetic field computation in the presence of thin shields with holes and the simple calculations needed to evaluate the shielding efficiency and the forces allow for a fast computer-assisted analysis and design. By comparison, the usual finite element or various hybrid methods are not recommended for field computations in systems containing thin shields, especially when the shields are multiply connected.

### VI. Conclusion

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### References