A Frequency-Domain Solution for the Motion of Levitated Conductors

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Abstract- Local coordinate systems are attached to the conducting bodies in motion which allows the usage of the eddy-current integral equation to determine the electromagnetic field. The space discretization is performed only for the conducting bodies and remains unchanged in time. An additional term, similar to the "motional" electromotive force in Faraday's law, is introduced in the integral equation. If the imposed currents are sinusoidal with time, a phasor representation is employed for a frequency-domain analysis based on the eddy-current integral equation. Since the "electrical" period is much smaller than the "mechanical" period of the body vibration, the time average force over an "electrical" period is used in the equation of motion. This allows for the solution of the equation of motion to be obtained by employing a time step which is greater than the "electrical" period. Thus, the efficiency of the method presented is much greater than the efficiency of the time-domain methods, where the time step is chosen to be a small fraction of the "electrical" period. This is more so at higher frequencies, which are used for more efficient levitation devices.

Keywords: eddy-current integral equation, electrodynamics of moving conductors, frequency-domain solution, levitation.

I. INTRODUCTION

The translational motion of a levitated solid conductor is described by the classical equation

\[ m \cdot \frac{d^2 r}{dt^2} = F(r, \frac{dr}{dt}, t) \]

where \( m \) is the mass, \( F \) is the resultant force that includes the magnetic force, \( r \) is the position vector of the center of gravity of the current distribution. For each position of the conductor, an electromagnetic field problem has to be solved in order to compute the force \( F \). At each time step, (1) is solved iteratively with a new value of \( r \) being obtained and the force being corrected for the subsequent iteration. This process is continued until the difference between two successive values of the force becomes sufficiently small, when the procedure is repeated for the next time step.

If the Finite Element Method is employed for the solution of the electromagnetic field problem, the discretization mesh has to be reconstructed at each iteration, which leads to a loss in accuracy and requires a large amount of computation. An important improvement was obtained by using a hybrid Finite Element – Boundary Element Method [1] for the electromagnetic computations, but the usage of the "laboratory" frame of reference complicates the field problem solution due to the presence of the motional electric field. The term \( \nu_0 \times B \) in Faraday's law that takes into account the motion, where \( \nu_0 \) is the velocity of the body in the "laboratory" frame of reference, vanishes when local frames of reference attached to the bodies in motion are used [2]-[4]. This also allows the usage of the simpler eddy-current integral equation for the bodies at rest, as it has been done in the case the velocities of the bodies are known [2], [4]. Usually, the velocities of the bodies are not known during the motion. In the case of electromagnetic levitation problems, in order to solve the coupled electro-mechanical problem and to ensure the stability of the solution, it is necessary to choose a sufficiently small time step (for example 200 time steps for one period at 50 Hz [1]). This leads to a huge computation effort, especially when the imposed current frequency increases.

In the present paper, the electromagnetic field problem is solved using the eddy-current integral equation. An additional term that takes into account the relative motion of the bodies is introduced in the integral equation. A frequency-domain solution is performed by employing a phasor representation. The time step used for the solution of (1) is much larger than that used in the time-domain techniques, especially at higher frequencies, for which an efficient technique is also presented for the acceleration of the overall computation process.

II. EDDY-CURRENT INTEGRAL EQUATION

In what follows, we consider two-dimensional field structures. The corresponding eddy-current integral equation for field problems with a solid conductor at rest is

\[ \rho J(r, t) + \gamma \frac{d}{dt} \int_{\Omega} J(r', t) \ln \frac{1}{R} dS' \]

\[ = -\gamma \frac{d}{dt} \int_{\Omega} J_s(r', t) \ln \frac{1}{R} dS' \]

where \( r \) and \( r' \) are the position vectors of the observation point and of the source point, respectively, \( \Omega \) is the region occupied by the solid conductor, \( \Omega_s \) is the region where the...
imposed current density $J_i$ is confined, $R = |r - r'|$, $\gamma = \frac{\mu_0}{2\pi}$, $\mu_0$ being the permeability of free space.

When the region $\Omega_i$ is moving with the velocity $\mathbf{v}$ in the frame of reference attached to $\Omega_i$, the eddy-current integral equation (2) becomes (see Appendix A)

$$\rho J_i(r, t) + \gamma \int \frac{\partial J_i(r', t)}{\partial t} dS' = -\gamma \int \frac{\partial J_i(r', t)}{\partial t} dS' - \int (\mathbf{n} \cdot \mathbf{v}) J_i(r', t) dS'$$

where $\mathbf{n}$ being the outwardly oriented normal unit vector.

III. PHASOR REPRESENTATION

For a "fixed" velocity $\mathbf{v}$, (3) is written in a phasor form as

$$\rho J_i(r) + j\omega J_i(r) + \gamma \int \frac{\partial J_i(r', t)}{\partial t} dS' = -j\omega \int \frac{\partial J_i(r', t)}{\partial t} dS' - \gamma \int (\mathbf{n} \cdot \mathbf{v}) J_i(r', t) dS'$$

where $\omega = 2\pi f$, $f$ being the frequency, and $j = \sqrt{-1}$. The first term on the right side of (4) represents the contribution to the induced electric field intensity due to the time variation of the imposed currents and the second one due to the relative motion.

For the numerical solution of (4), the region $\Omega$ is divided into polygonal surface elements $\omega_m$, within which the induced current density is considered to be constant. The region $\Omega_i$ is divided into surface elements $\omega_{mk}$, with the imposed current density being constant. Integrating (4) over each $\omega_m$ yields the following matrix equation for the vector of the unknown induced current density:

$$AJ^{re} - \lambda B J^{im} = \lambda B J^{re} - \gamma C J^{im}$$

$$AJ^{im} + \lambda B J^{re} = -\lambda B J^{re} - \gamma C J^{im}$$

the superscripts "re" and "im" indicating the real and the imaginary part of the phasors $J$ and $J_i$, respectively. $A$ is a diagonal matrix with entries $A_{mk} = \rho_m S_m$, $\rho_m$ being the resistivity of the material in $\omega_m$ and $S_m$ its area, and $B$ is a symmetric matrix whose entries are

$$B_{mk} = \int \int \ln \frac{1}{R} \frac{1}{R} dS_k dS_m = S_m S_k$$

$$-\frac{1}{4} \int \int (\mathbf{n}_m \cdot \mathbf{n}_k) R^2 \ln \frac{1}{R} dS_k dS_m .$$

The entries of the matrix $B_i$ are defined as in (6), but with the elements $a_i$ of $\Omega$ being replaced with the elements $a_{mk}$ belonging to $\Omega_i$, while the entries of the matrix $C$ are

$$C_{mk} = \frac{1}{2} \int \int (\mathbf{n}_m \cdot \mathbf{R})(\mathbf{n}_k \cdot \mathbf{v}) \ln \frac{1}{R} dS_k dS_m .$$

All integrals in (6) and (7) are evaluated using analytic expressions. The entries of the matrix $B_i$ and $C$ are to be computed for each new position of $\Omega_i$.

The average magnetic force over a period is evaluated using the expression

$$F_{av} = -\gamma \Re 2 \Omega \Omega_i \int \int \{J(r') J_i(r') \cdot \frac{r-r'}{|r-r'|^2} dS_i dS'$$

where the asterisk indicates the complex conjugate.

IV. ITERATIVE SOLUTION OF THE EQUATION OF MOTION

Equation (1) is solved iteratively. We choose an appropriate time step $\Delta t = n T$, where $n = 1, 2, 3, \ldots$ and $T$ is the period of the imposed current, and assume that the magnetic force $F_{av}$ has a linear variation during $\Delta t$. At the time $t$ the body has a position defined by the position vector $\mathbf{r}_0$ and a magnetic force $F_{av0}$ is exerted upon it. The iterative process is started by imposing the value $F_{av1} = F_{av0}$ at the time $t + \Delta t$ and the position vector $\mathbf{r}_0$ results from solving (1). The electromagnetic field problem is then solved for the new $\mathbf{r}_1$ and a new value of the force $F_{av1}$ is computed for the time $t + \Delta t$. This operation is repeated until the difference between two successive values of the magnetic force for the time $t + \Delta t$ is sufficiently small and, then, we proceed to the next time step.

To accelerate substantially the solution of the equation of motion, the time step is chosen to be a multiple $n$ of the "electrical" period which is adjusted according to the force value, such that when the force decreases the time step is increased and when the force increases it is reduced (see Figs. 7 and 8).

V. ILLUSTRATIVE EXAMPLE

A copper plate (see Fig. 1) of width 80 mm, thickness 4 mm, resistivity $2 \times 10^{-8} \Omega m$ and of mass density $8.9 \times 10^3 \text{kg/m}^3$ is levitated using two coils of 200 turns each, of 10 mm x 10 mm in cross section and a distance between the axes of their sides of 70 mm and 30 mm, respectively. The current direction is the same in the outer and inner coils, the current intensity in each turn being $i = 1 \sqrt{2} \sin (2\pi t)$, with $I= 10 \text{A}$ [5].
Initially, the conducting plate is located at 10 mm above the coils ($y=0$). It is assumed that the plate only moves in the vertical direction, but the procedures described in the paper are also applicable when more degrees of freedom are considered. The plate cross section is discretized in 180 rectangular elements, as indicated in Fig. 1. The motion of the levitated plate for $f=200$ Hz is presented in Fig. 2. With a constant time step equal to one “electrical” period, the overall computation time of the results in Fig. 2 was about 75 s, employing a 2.128 GHz Intel processor notebook. The results for the first second are shown in Fig. 3. When a variable time step is used the computation time decreases to 24 s and the results (shown in Fig. 4) are practically the same as those in Fig. 3.

The efficiency of the method presented increases spectacularly with frequency. Results for $f=1,000$ Hz are presented in Fig. 5 for the first 13 s. For the first second the evolution is shown in Fig. 6. The results in Figs. 5 and 6 have been obtained using a variable time step, as indicated in Figs. 7 and 8. A computation time of 50 s was required to generate...
the results in Fig. 5. The results in Fig. 5 were also obtained by applying the time-domain method presented in [5], where the "electrical" period was divided into 48 equal time steps, the computation time required being 10,848 s.

VI. CONCLUSIONS AND REMARKS

An efficient method is presented for computing the motion of solid conductors under the action of electromagnetic forces. For practically the same accuracy of the results, a tremendous reduction in the amount of computation is achieved when using the proposed frequency-domain procedure, instead of the usual time-domain method. The new procedure is particularly useful at high frequencies which are employed for more efficient levitation devices, where the "electrical" period is much smaller than the "mechanical" period.

The method can be extended to nonlinear media, using the procedures proposed in [6]-[8]. Since in some problems, for instance in electromagnetic levitation problems, the air regions are relatively large with respect to the conducting and/or ferromagnetic regions, the weight of the fundamental harmonic in the harmonic spectrum is significant and, thus, for the convergence acceleration in the polarization method one can efficiently employ the technique proposed in [9].

APPENDIX A
DERIVATION OF THE EQUATION (3)

The time derivative of the last integral in (2), when the region \( \Omega_i \) is moving with a velocity \( v \), is evaluated as

\[
\frac{d}{dt} \int_{\Omega_i(t)} J_i(r',t) \ln \frac{1}{R(t)} dS' = \lim_{\Delta t \to 0} \left[ \int_{\Omega_i(t+\Delta t)} J_i(r',t+\Delta t) \ln \frac{1}{R(t+\Delta t)} dS' - \int_{\Omega_i(t)} J_i(r',t) \ln \frac{1}{R(t)} dS' \right]
\]

\[
= \lim_{\Delta t \to 0} \left[ \int_{\Omega_i(t+\Delta t)} \frac{1}{\Delta t} \left( J_i(r',t+\Delta t) \ln \frac{1}{R(t+\Delta t)} - J_i(r',t) \ln \frac{1}{R(t)} \right) dS' \right]
\]

\[
+ \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left( \int_{\Omega_i(t+\Delta t)} - \int_{\Omega_i(t)} J_i(r',t) \ln \frac{1}{R(t)} dS' \right)
\]

\[
= \int_{\Omega_i(t)} \frac{\partial}{\partial t} \left( J_i(r',t) \ln \frac{1}{R(t)} \right) dS' + \int_{\partial \Omega_i(t)} (n \times v) J_i(r',t) \ln \frac{1}{R} d\sigma.
\]

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