TRANSIENT PROPAGATION ALONG CORONATING MONOPOLAR TRANSMISSION LINES

D.V. Brankovic  M.R. Raghveer  I.M.R. Ciric
Electrical Engineering Department
University of Manitoba
Winnipeg, Manitoba
R3T 2N2
Canada

Introduction

High Voltage transmission line corona and its effects have been studied since the beginning of this century. Peck's pioneering work in the area /1/ resulted in empirical expressions for corona inception voltage and loss; these expressions are in use even today. Peck's work was followed by that of many other researchers. Ryan and Henline /2/ explained the hysteresis character of corona and Skilling and Dykes /3/ published experimental results showing the influence of corona on the distortion of voltage surges travelling along a power line. Later, Wagner and Lloyd /4/ showed that the charge-voltage (QV) curves, resulting from the application of impulse voltages to a power line, have the form of a loop a typical example of which is shown in Fig. 1.

It can be noted from this figure that the section OA is the only linear portion of the curve; this segment corresponds to non corona conditions. The variable slope of the QV loop corresponds to the capacitance term in the transmission line equations and makes these equation non linear. Furthermore, the area enclosed by such a curve is proportional to the energy dissipated due to corona.

Due to an increase in interest in transmission line transient phenomena, the effect of corona on travelling waves was studied extensively during the last decade /5,6,7,8,9,10/ and various mathematical models have been proposed. These models differ from one another in the manner in which the non linear QV relationship is taken into account and in the numerical procedures adopted to solve the non linear system of transmission line equations.

In this paper three methods of modelling the effect of corona on the propagation of transients are compared with a view to assessing their applicability and accuracy. At the same time an efficient and straightforward finite difference based algorithm has been used resulting in substantially reduced computation times.

Corona models for transmission line transients

There are two basic approaches to model impulse corona. One approach is based on analytical approximations of experimentally recorded QV curves for different conductor, bundle configurations, weather conditions and different shapes and polarities of applied voltage impulses. These analytical expressions, proposed by different authors, are dependant on several empirical constants which account for the difference in the shape of QV curves under different practical situations. Since the area enclosed by a QV curve accounts for corona losses, the conductance term is excluded from the transmission line equations which are expressed in the following form:

\[ \frac{\partial Q}{\partial t} = \left( \frac{\partial Q}{\partial V} \right) \left( \frac{\partial V}{\partial t} \right) \]

\[ \frac{\partial V}{\partial x} = L_0 \left( \frac{\partial I}{\partial t} \right) + R_0 J \]

where \( L_0 \) and \( R_0 \) are the self inductance and resistance per unit length of the line respectively and \( \partial Q/\partial V \) represents the dynamic capacitance per unit length of the line. The models based on Eq. 1 are called QV loop models.

The other approach is based on considering only that part of an experimental QV curve where the first derivative is positive; corona losses are taken into account by an appropriate conductance term derived from existing corona loss laws. The equations in this case have the following form.

\[ \frac{\partial I}{\partial x} = \left( \frac{\partial Q}{\partial V} \right) \left( \frac{\partial V}{\partial t} \right) + GV \]

\[ \frac{\partial V}{\partial x} = L_0 \left( \frac{\partial I}{\partial t} \right) + R_0 J \]

Fig. 1. Typical QV Loop
where G is the equivalent conductance and $\partial Q/\partial V$ is determined from the approximate rising section of the QV loop. The models based on Eq. 2 are called conductance models.

Three models were chosen for comparison: the conductance model proposed by Lee /6/ and the QV loop models proposed by Gary et al. /7/ and Inoue /9/.

In the conductance model /6/ the dynamic capacitance in Eq. 2 is evaluated in terms of one empirical parameter and the voltage dependent conductance term is derived from consideration of Peek's quadratic loss law by involving another empirical parameter. Both parameters are chosen to give good agreement between simulated results and field measurements obtained from a test line.

The model used in /7/ includes both corona and skin effects. This latter effect was excluded from our present study since it is not considered in the other two models. In this model the dynamic capacitance, $\partial Q/\partial V$, in Eq. 1, is expressed in terms of two empirical parameters derived such that the QV loops are best fitted with laboratory obtained experimental loops for a large variety of conductors and for impulses with positive polarity.

The model used in /9/ also utilizes QV loops; however, the dynamic capacitance term in Eq. 1 is modelled in terms of five parameters which are chosen empirically to obtain the best possible agreement between the calculated and measured voltage waveforms for positive and negative polarities, various geometrical configurations and sending end voltage magnitudes. As in /6/ experimental data from a test line is used to achieve this condition.

All the three models considered are time domain based and rely on empirical constants which are, strictly speaking, valid for a particular geometrical configuration of a power line.

In this paper the above modelling procedures are compared by using the Finite Difference method. Moreover, in the application of the method the combination of time and displacement steps were varied in order to obtain the shortest possible computation time while satisfying the stability condition /11/ and yet maintaining a high accuracy.

Comparative analysis and results

The comparative study was carried out by applying each of the three modelling procedures to a test line considered in both /6/ and /9/. The test line configuration consists of a single conductor line of radius 12.65 mm strung at an average suspension height of 22.2 m.

Experimental results for the test line are available in each of references 6 and 9 for different sending end lightning impulse voltage levels. In both references the experimental data was obtained with no steady state voltage on the line. This data was used to establish the accuracy of the modelling techniques.

In the application of the methods of /6/ and /9/ to this line, the values of the empirical constants used in the numerical computations were identical to those recommended by the authors of these methods. The line considered in /7/ is different from that in /6/ and /9/. Therefore empirical constants suggested by the author in /7/ cannot be used when employing the QV loop method of /7/ in the analysis of the test line considered in /6/ and /9/. In this case the two empirical parameters were varied over a suitable range and chosen so that best agreement could be obtained between simulated and experimental results.

First, the input waveform used in /6/, Fig. 2, was considered for all the three models. Voltage waveforms were computed at distances of 352.5, 705 and 1057.5 m from the sending end of the line and the results were compared with the experimental data reported in /6/. The accuracy of the results of the simulation was assessed by comparing the standard deviations computed as follows

\[
\text{Standard deviation} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (V(x,i) - V_b(x,i))^2}
\]

where

- \(V(x,i)\) = values of voltage computed at distance \(x\) and time \(i\)
- \(V_b(x,i)\) = experimental value measured at the same location and time
- \(N\) = number of discrete values of voltage (10 per µs)

The standard deviations were computed in three time ranges A, B and C which covered times 0 to 2 µs, 2 to 3 µs and 3 to 4 µs respectively. The computed standard deviation values are shown in Table I and the experimental and computed voltage waveforms appear in Figs. 3-5.

### Table I: Standard deviations, in kV, for the 3 models computed using the experimental results in /6/ as basis

<table>
<thead>
<tr>
<th>Conductance Model /6/</th>
<th>X=352.5m</th>
<th>X=705m</th>
<th>X=1057.5m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Range A</td>
<td>52.75</td>
<td>59.29</td>
<td>80.82</td>
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<tr>
<td>Time Range B</td>
<td>36.72</td>
<td>24.42</td>
<td>87.38</td>
</tr>
<tr>
<td>Time Range C</td>
<td>73.27</td>
<td>101.71</td>
<td>246.80</td>
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</table>

<table>
<thead>
<tr>
<th>Q-V loop model /9/</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Range A</td>
<td>64.12</td>
<td>73.30</td>
<td>78.60</td>
</tr>
<tr>
<td>Time Range B</td>
<td>57.67</td>
<td>111.03</td>
<td>43.94</td>
</tr>
<tr>
<td>Time Range C</td>
<td>50.28</td>
<td>60.71</td>
<td>258.52</td>
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</table>

<table>
<thead>
<tr>
<th>Q-V loop model /7/</th>
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</thead>
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<tr>
<td>Time Range A</td>
<td>105.87</td>
<td>118.26</td>
<td>126.13</td>
</tr>
<tr>
<td>Time Range B</td>
<td>112.45</td>
<td>210.87</td>
<td>335.18</td>
</tr>
<tr>
<td>Time Range C</td>
<td>25.10</td>
<td>81.87</td>
<td>469.56</td>
</tr>
</tbody>
</table>

Fig. 2. Input waveform used in /6/

Fig. 3. Comparison between waveforms computed by the conductance model /6/ and experimental waveforms in /6/.

--- experimental --- computed
A similar comparison was carried out by considering the input waveform used in /9/, Fig. 6, for each of the three models. The voltages were computed at the same distances as in the previous case and compared with the experimental values reported in /9/. The standard deviations computed according to Eq. 3 are shown in Table II; the experimental and computed voltage waveforms are shown in Figs. 7–9.

Table II: Standard deviations, in kV, for the 3 models computed using the experimental results in /9/ as basis

<table>
<thead>
<tr>
<th>Conductance Model /6/</th>
<th>X=352.5m</th>
<th>X=705m</th>
<th>X=1057.5m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Range A</td>
<td>67.55</td>
<td>69.40</td>
<td>68.29</td>
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<tr>
<td>Time Range B</td>
<td>46.46</td>
<td>76.14</td>
<td>54.50</td>
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<tr>
<td>Time Range C</td>
<td>44.32</td>
<td>65.24</td>
<td>95.98</td>
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Table II: (continued)

<table>
<thead>
<tr>
<th>Model</th>
<th>Time Range A</th>
<th>Time Range B</th>
<th>Time Range C</th>
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<tr>
<td>Q-V loop</td>
<td>116.28</td>
<td>50.87</td>
<td>59.79</td>
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<tr>
<td>model /9/</td>
<td>70.99</td>
<td>112.53</td>
<td>116.40</td>
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<tr>
<td>Tune</td>
<td>115.39</td>
<td>129.53</td>
<td>141.49</td>
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<table>
<thead>
<tr>
<th>Model</th>
<th>Time Range A</th>
<th>Time Range B</th>
<th>Time Range C</th>
</tr>
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<tbody>
<tr>
<td>Q-V loop model /7/</td>
<td>98.26</td>
<td>65.74</td>
<td>62.05</td>
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<td></td>
<td>185.51</td>
<td>303.56</td>
<td>140.32</td>
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<tr>
<td></td>
<td>159.24</td>
<td>386.99</td>
<td>387.76</td>
</tr>
</tbody>
</table>

The results of the comparison shown in Figs. 3–5, 7–9 and Tables 1 and 2 indicate that the model with the highest accuracy is the conductance model. Slightly less accurate results were obtained with the QV loop model of /9/ and much less accurate results with the QV loop model of /7/. This is probably due to the fact that the former QV loop model uses five empirical parameters, instead of two in the latter model, to describe the corona influence on the QV loops. Furthermore, the five constants in the former method are chosen by comparison with experimental voltage waveforms.

The rather surprising result that the conductance model yields the best results can be explained as follows. First, the QV loop methods do not take into account the dependence of the size and shape of the loop on the location along the line. Secondly the conductance model relies on a well established loss law. Moreover the two empirical constants in this method are chosen by comparison with experimental voltage waveforms.

Conclusions

Three present day models of incorporating the corona effect in the propagation of transients along a power line have been compared with regard to accuracy. The results clearly indicate that the simpler conductance model yields the best results. All the models considered rely on empirically obtained parameters and therefore can not be extended easily to cover other cases for which experimental results are not available.

The accuracy and applicability of the three methods are assessed by implementing them using an optimized finite difference approach which greatly improved solution efficiency.

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References


Address of main author
D.V. Brankovic
University of Manitoba
Winnipeg, MB
Canada R3T 2N2