1. INTRODUCTION

In the last decade, considerable attention has been devoted to the analysis of time-domain electromagnetic scattering problems. The time-domain current and field response of a circular cylinder to a plane electromagnetic wave has been already studied by many researchers [1]-[4]. Recently, the transient response of a conducting cylinder to a cylindrical electromagnetic wave has also been studied by the authors [5]-[7]. The purpose of the present paper is to derive analytical expressions of the time-domain field response of a dielectric circular cylinder to a cylindrical electromagnetic wave generated by a parallel filament carrying a unit-step current. The mathematical technique employed here is similar to that in [7]. The time-domain expression of the electric field is obtained by the inverse Laplace transform of the frequency-domain eigenfunction solution. The response contains contributions of the branch cut integrals and residues corresponding to appropriately chosen integration contours.

2. THE FREQUENCY-DOMAIN SOLUTION

The geometry of the problem is shown in Fig. 1 where \( a \) is the radius of the dielectric cylinder with permittivity \( \varepsilon = \varepsilon_r \varepsilon_0 \), permeability \( \mu = \mu_0 \), and \( r_0 \) the distance from the line source to the cylinder axis. The surrounding medium is assumed to be free space with permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \). For a line source carrying a unit-step current, the Laplace transform of the electric field in the absence of the cylinder is

\[
E_z^i(r, \phi, s) = -\frac{\mu_0}{2\pi} \left\{ \sum_{n=0}^{\infty} \varepsilon_n I_n \left( \frac{sr}{c} \right) K_n \left( \frac{sr_0}{c} \right) \cos n \phi \right\}
\]

where \( \varepsilon_n = 1 \) for \( n = 0 \) and \( \varepsilon_n = 2 \) for \( n \neq 0 \). \( r \) and \( \phi \) are the cylindrical coordinates of the field point, \( s \) the Laplace transform variable and \( c \) the speed of light. \( I_n \) and \( K_n \) are the modified Bessel functions of the first and second kind, respectively. The Laplace transform of the scattered electric field outside the
cylinder can be assumed as

\[ E_z^s(r, \phi, s) = -\frac{\mu_0}{2\pi} \sum_{n=0}^{\infty} \varepsilon_n a_n K_n \left( \frac{sr}{c} \right) K_n \left( \frac{sr_0}{c} \right) \cos n \phi \] (2)

and the Laplace transform of the electric field inside the cylinder as

\[ E_z(r, \phi, s) = -\frac{\mu_0}{2\pi} \sum_{n=0}^{\infty} \varepsilon_n b_n I_n \left( \frac{sr}{\lambda c} \right) \cos n \phi \] (3)

where \( \lambda = \sqrt{\epsilon} \). The magnetic fields corresponding to \( E_1^i, E_2^i \) and \( E_z \) are

\[
H_z^i(r, \phi, s) = -\frac{1}{2\pi c} \sum_{n=0}^{\infty} \varepsilon_n a_n K_n \left( \frac{sr}{c} \right) K_n \left( \frac{sr_0}{c} \right) \cos n \phi
\quad r < r_0
\]

(4)

\[
H_z^s(r, \phi, s) = -\frac{1}{2\pi c} \sum_{n=0}^{\infty} \varepsilon_n a_n K_n \left( \frac{sr}{c} \right) K_n \left( \frac{sr_0}{c} \right) \cos n \phi
\quad r > r_0
\]

(5)

\[ H_z(r, \phi, s) = -\frac{\lambda}{2\pi c} \sum_{n=0}^{\infty} \varepsilon_n b_n I_n \left( \frac{sr}{\lambda c} \right) \cos n \phi \] (6)

Using the boundary condition at \( r = a \), the constants \( a_n \) and \( b_n \) can be obtained as

\[
a_n = \frac{I_n \left( \frac{sa}{c} \lambda \right) I_{n+1} \left( \frac{sa}{c} \lambda \right) - \lambda I_n \left( \frac{sa}{c} \lambda \right) I_{n+1} \left( \frac{sa}{c} \lambda \right)}{I_n \left( \frac{sa}{c} \lambda \right) K_{n+1} \left( \frac{sa}{c} \lambda \right) + \lambda K_n \left( \frac{sa}{c} \lambda \right) I_{n+1} \left( \frac{sa}{c} \lambda \right)} \] (7)

\[
b_n = \frac{1}{sa/c} \frac{K_n \left( \frac{sr_0}{c} \right)}{I_n \left( \frac{sa}{c} \lambda \right) K_{n+1} \left( \frac{sa}{c} \lambda \right) + \lambda K_n \left( \frac{sa}{c} \lambda \right) I_{n+1} \left( \frac{sa}{c} \lambda \right)} \] (8)

From (2), (3), (7) and (8), we obtain the frequency-domain solution of the scattered electric field outside the cylinder and the total electric field inside the cylinder, respectively, as

\[ E_z^s(r, \phi, s) = -\frac{\eta_0}{2\pi c} \sum_{n=0}^{\infty} \varepsilon_n \cos n \phi \frac{I_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right) - \lambda I_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right)}{I_n \left( \frac{\zeta \lambda}{c} \right) K_{n+1} \left( \frac{\zeta \lambda}{c} \right) + \lambda K_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right)} \] (9)

and

\[ E_z^s(r, \phi, s) = -\frac{\eta_0}{2\pi c} \sum_{n=0}^{\infty} \varepsilon_n \cos n \phi \frac{I_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right) - \lambda I_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right)}{I_n \left( \frac{\zeta \lambda}{c} \right) K_{n+1} \left( \frac{\zeta \lambda}{c} \right) + \lambda K_n \left( \frac{\zeta \lambda}{c} \right) I_{n+1} \left( \frac{\zeta \lambda}{c} \right)} \] (9)
where \( \zeta = \frac{sa}{c}, r' = r/a, r_0' = r_0/a \) and \( \eta_0 \) is the characteristic impedance of free space.

3. THE TIME-DOMAIN SOLUTION

The inverse Laplace transform of (10) can be written as

\[
E_z(r, \phi, t) = \frac{\eta_0}{2\pi a} \sum_{n=0}^{\infty} e_n \cos \phi \frac{I_n(\zeta r \lambda) K_n(\zeta r_0)}{I_n(\zeta \lambda) K_{n+1}(\zeta) + \lambda K_n(\zeta) I_{n+1}(\zeta \lambda)}
\]

(10)

where \( \zeta = \frac{c t}{\alpha} \) and

\[
h_n(\tau) = \frac{1}{2\pi j} \int_{C_{\infty}} H_n(\zeta) e^{\zeta \tau} d\zeta
\]

(12)

in which

\[
H_n(\zeta) = \frac{1}{\zeta} \frac{I_n(\zeta r \lambda) K_n(\zeta r_0)}{I_n(\zeta \lambda) K_{n+1}(\zeta) + \lambda K_n(\zeta) I_{n+1}(\zeta \lambda)}
\]

(13)

The appropriate Bromwich contour and associated integration contours in the complex \( \zeta \) plane are shown in Fig. 2. When the contour is closed in the right half plane, we have

\[
h_n(\tau) = 0 \quad \text{for} \quad \tau < (r_0' - 1) + \lambda (1 - r')
\]

(14)

since it can be shown that \( f_n(\zeta) = I_n(\zeta \lambda) K_{n+1}(\zeta) + \lambda K_n(\zeta) I_{n+1}(\zeta \lambda) \) has no zeros in the right half plane and the contour integral along \( C_{\infty} \) vanishes. When the contour is closed along \( C_{\infty} + L_1 + C_\epsilon + L_2 \) in the left half plane, \( h_n(\tau) \) can be written as

\[
h_n(\tau) = -\frac{1}{2\pi j} \left[ \int_{C_\epsilon} + \int_{L_1} + \int_{L_2} \right] H_n(\zeta) d\zeta + \sum R_{nl}
\]

(15)

where \( R_{nl} \) is the residue of \( H_n(\zeta) e^{\zeta \tau} \) at the pole \( \zeta_{nl} \) at which \( f_n(\zeta) = 0 \). It can be shown that

\[
\lim_{R_{\infty} \to 0} \int_{C_\epsilon} H_n(\zeta) e^{\zeta \tau} d\zeta = 0
\]

(16)

and, when \( \tau > (r_0' - 1) + \lambda (1 - r') \),
Using appropriate analytic continuation of the modified Bessel functions, the contributions from the line integrals along the branch cut can be evaluated as

\[ \int_{L_1} H_n(\zeta) e^{\zeta \tau} d\zeta + \int_{L_2} H_n(\zeta) e^{\zeta \tau} d\zeta = (-1)^n 12\pi j B_n(\tau) \]

where

\[ B_n(\tau) = \int_0^\infty \frac{e^{-\frac{nr}{\tau}}}{z} \frac{I_n(zr'\lambda) [K_n(zr'_0)B + I_n(zr'_0)A]}{A^2 + \pi^2 B^2} \, dz \]

in which

\[ A = I_n(z\lambda) K_{n+1}(z) + \lambda I_{n+1}(z\lambda) K_n(z) \]

\[ B = I_n(z\lambda) I_{n+1}(z) - \lambda I_{n+1}(z\lambda) I_n(z) \]

For the residue terms, since the zeros of \( f_n(\xi) \) appear in conjugate pairs, \( R_n \) can be obtained as

\[ R_n = 2\text{Re} \left[ \frac{e^{\xi \omega \tau}}{\xi} \frac{I_n(\lambda r'\xi_{nl}) K_n(r_0'\xi_{nl})}{\lambda^2 I_{n+2}(\lambda \xi_{nl}) K_n(\xi_{nl}) - I_n(\lambda \xi_{nl}) K_n(\lambda \xi_{nl})} \right] \]

and the summation over \( l \) in (15) will be up to \( l = \lceil \text{number of zeros for each } n \rceil / 2 \). From (11)–(18), we finally obtain

\[ E_z(r,\phi,t) = -\frac{\eta_0}{2\pi a} \sum_{n=0}^\infty \xi_n \cos n\phi \left[ (-1)^n B_n(\tau) + \sum_l R_l \right] \text{ for } \tau > (r'_0 - 1) + \lambda (1 - r') \]

with \( B_n(\tau) \) and \( R_n \) given by (19) and (22), respectively. By following the same procedure, we can obtain the time-domain solution of the scattered electric field outside the cylinder as

\[ E^t_z(r,\phi,t) = -\frac{\eta_0}{2\pi a} \sum_{n=0}^\infty \xi_n \cos n\phi \left[ (-1)^n B^t_n(\tau) + \sum_l R^t_l \right] \text{ for } \tau > r'_0 + r' \]

where

\[ R^t_l = -2\text{Re} \left[ \frac{\lambda e^{\xi \omega \tau}}{\xi_{nl}} \frac{I_n(\lambda \xi_{nl})}{K_{n+1}(\xi_{nl})} \frac{K_n(r'\xi_{nl}) K_n(r'_0\xi_{nl})}{\lambda^2 I_{n+2}(\lambda \xi_{nl}) K_n(\xi_{nl}) - I_n(\lambda \xi_{nl}) K_n(\lambda \xi_{nl})} \right] \]

and

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\[ B_n(t) = \frac{1}{2} \int_0^\infty \frac{B [2AD + BC]}{A^2 + \pi^2 B^2} e^{-\zeta t} \, d\zeta \]  \hspace{1cm} (26)

in which

\[ C = K_n(\zeta r') K_n(\zeta r_0') - \pi^2 l_n(\zeta r') l_n(\zeta r_0') \]  \hspace{1cm} (27)

and

\[ D = l_n(\zeta r') K_n(\zeta r_0') + l_n(\zeta r_0') K_n(\zeta r') \]  \hspace{1cm} (28)

DISCUSSION

The analytical expressions of the time-domain electric fields inside and outside the dielectric cylinder are derived. It should be noted that for any \( \phi \), the expression of the total electric field inside the cylinder is valid for all the time ranges while that of the scattered field outside the cylinder is valid only after the initial incident wave front has passed a distance of \( r_0 + r \). In order to obtain numerical results from (23) and (24), one should be able to calculate the zeros of \( f_n(\zeta) \) in the left half \( \zeta \)-plane. This is currently under investigation.

REFERENCES


Fig. 1. Cross section of the dielectric cylinder to line source configuration.

Fig. 2. The integration contour.