SCATTERING FROM TWO CONDUCTING SPHERES COVERED WITH A DIELECTRIC LAYER

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INTRODUCTION

Analytic solution to the scattering of a plane electromagnetic wave by two conducting spheres covered with a dielectric layer is formulated using the multiple expansion method and the translation addition theorem for the vector spherical wave functions. Numerical results are presented to show the effects of the dielectric coating on the scattering cross section behaviour.

The scattering of a plane electromagnetic wave by one sphere covered with a dielectric layer has been studied by many authors. Aden and Kerker (1) obtained analytic expressions to the scattering of plane electromagnetic waves by a dielectric-coated sphere with a concentric spherical shell of a different dielectric material, while Scharfman (2) presented numerical values for the special case of a small (kR<1) dielectric-coated conducting sphere. It was found out from these studies that the presence of dielectric coatings could lead to substantial increase in the backscattering cross section for an appropriate choice of the dielectric constant and the thickness of the coating with respect to that for an uncoated sphere. This is due to the multiple field interactions within the dielectric coating.

Up to now there is no analytical treatment of the scattering of plane electromagnetic waves by a system with more than one sphere covered with a dielectric layer. In this paper we study the effect of dielectric-coating on the multiple scattering behaviour for a system of two conducting spheres, since the solution to this problem is a first step towards the more general solution for N dielectric-coated spheres which has potential applications in the simulation of complex bodies and in the loading of aperture antennas. Analytic solution to the scattering by two dielectric spheres was obtained by Bruning and Lo (3), while numerical results based on the moment method were reported in references (4) and (5). Recently, the authors derived analytic and also approximate solutions for the scattering of plane electromagnetic waves by an arbitrary configuration of conducting (6) or dielectric (7) spheres.

FORMULATION

Fig. 1 shows the system geometry. The spheres A and B are spaced along the z-axis and centered at the origins O and O', respectively. The separation distance between the centers of the spheres is d. The incident wave has a unit electric field intensity and a propagation vector k which makes an angle α with the positive z-direction. The incident electric field is assumed to be polarized in the f direction and has the form

\[ E_i = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left( P(m,n) \tilde{E}_m^{(1)}(r,\theta,\phi) + Q(m,n) \tilde{N}_m^{(2)}(r,\theta,\phi) \right) \]

where \( P \) and \( Q \) are the incident field expansion coefficients, while \( \tilde{E}_m^{(1)} \) and \( \tilde{N}_m^{(2)} \) are the spherical vector wave functions of the first kind given by

\[ \tilde{E}_m^{(1)}(r,\theta,\phi) = \frac{1}{k} \mathbf{v} \times \tilde{N}_m^{(1)}(r,\theta,\phi) \]

\[ \tilde{N}_m^{(1)}(r,\theta,\phi) = \frac{1}{k} \mathbf{v} \times \tilde{N}_m^{(0)}(r,\theta,\phi) \]

in which

\[ \mathbf{v} = (0,0,1) \]

Here \( \mathbf{v} \) is the vector Bessel function and \( P_m^\alpha \) the associated Legendre function of the first kind. The expansion coefficients in equation (3) may be obtained by using the orthogonality properties of the vector wave functions as in reference (8), i.e.,

\[ P(m,n) = \int_0^\pi \int_0^{2\pi} \tilde{E}_m^{(1)}(r,\theta,\phi) \tilde{E}_m^{(1)*}(r,\theta,\phi) \sin \theta \ d\theta \ d\phi \]

\[ Q(m,n) = \int_0^\pi \int_0^{2\pi} \tilde{N}_m^{(1)}(r,\theta,\phi) \tilde{N}_m^{(1)*}(r,\theta,\phi) \sin \theta \ d\theta \ d\phi \]

For the limiting case where the direction of the incident plane wave coincides with the positive z-axis (\( \theta=0 \)), equations (7) and (8) reduce to

\[ P(m,n) = Q(m,n) = \int_0^\pi \int_0^{2\pi} \tilde{E}_m^{(1)}(r,\theta,\phi) \tilde{E}_m^{(1)*}(r,\theta,\phi) \sin \theta \ d\theta \ d\phi \]

\[ \delta_{n,1} \]

being the Kronecker delta. The scattered field form sphere A (\( r>a \)) can be written in terms of the spherical vector wave functions as

\[ \tilde{E}_s = \sum_{m=-n}^{\infty} \sum_{n=0}^{\infty} \left( A_s(m,n) \tilde{H}_m^{(1)}(r,\theta,\phi) + B_s(m,n) \tilde{N}_m^{(2)}(r,\theta,\phi) \right) \]

Here \( A_s \) and \( B_s \) are the scattering coefficients of sphere A for transverse magnetic (TM) and transverse electric (TE) waves, while \( \tilde{H}_m^{(1)} \) and \( \tilde{N}_m^{(2)} \) are the vector spherical wave functions of the third kind and may be obtained by replacing \( J_n \) by the spherical Hankel function \( \tilde{h}_n^{(1)} \) in equation (6).

The fields in region I (\( 0<r<a \)) are expressed in terms of the vector spherical wave functions of the first and third kinds. Hence the electric field can be written as

\[ \tilde{E}_I = \sum_{m=-n}^{\infty} \sum_{n=0}^{\infty} \left( A_I(m,n) \tilde{H}_m^{(1)}(r,\theta,\phi) + B_I(m,n) \tilde{N}_m^{(2)}(r,\theta,\phi) \right) \]

The scattered field from sphere A in the presence of sphere B is due to the incident field and to the outgoing scattered field from sphere B. In order to impose the boundary conditions at \( r=a \), the latter scattered field is transformed into an incoming field with respect to the sphere A, expressed in terms of the coordinates attached to this sphere. The boundary conditions require continuity of the tangential components (\( \theta \) and \( \phi \)) of the electric and magnetic fields, i.e.,
Since the equations for the unknown field scattering coefficients in the theorem, we present here expressions for the scattering coefficients in a similar way.

\[ E_{m}^{(0)} = \sum_{n} (b_{m,n}^{(0)} P_{m,n}(\cos \theta) + b_{m,n}^{(1)} P_{m,n}(\cos \theta)) \]  

\[ A_{m,n}^{(0)} = a_{m,n}^{(0)} P_{m,n}(\cos \theta) + a_{m,n}^{(1)} P_{m,n}(\cos \theta) \]

where \( A_{m,n}^{(0)} \) and \( B_{m,n}^{(0)} \) are the translation coefficients in the addition theorem, while \( B_{m,n}^{(0)} \) and \( B_{m,n}^{(1)} \) are the scattering coefficients of sphere B. The boundary conditions at \( n_0 \) and \( n_2 \) are implemented in a similar way.

Since we are mainly interested in the field outside the spheres, we present here expressions for the scattering coefficients in equations (10) and (15). Using the orthogonality properties of the vector wave functions leads to a system of coupled linear equations for the unknown field scattering coefficients in the form

\[ A_{m,n}^{(0)} = a_{m,n}^{(0)} P_{m,n}(\cos \theta) + a_{m,n}^{(1)} P_{m,n}(\cos \theta) \]

\[ A_{m,n}^{(1)} = a_{m,n}^{(1)} P_{m,n}(\cos \theta) + a_{m,n}^{(1)} P_{m,n}(\cos \theta) \]

where \( P' \) and \( Q' \) are the incident field expansion coefficients relative to the sphere B, which differ from those relative to the sphere A by the phase factor \( e^{i \theta_{AB}} \), while \( v_{m,n} \) and \( v_{m+1,n} \) are the scattering coefficients corresponding to the scattering in the conductive sphere A covered with a dielectric layer, assumed to be alone in the incident field, which are given by

\[ v_{m,n} = -\frac{a_{m,n}^{(0)}}{a_{m,n}^{(1)}} - \sum_{l} (1-l)^{m+n} \sin \theta A_{m,n}^{(0)}(\cos \theta) \]

\[ v_{m+1,n} = -\frac{a_{m+1,n}^{(0)}}{a_{m+1,n}^{(1)}} - \sum_{l} (1-l)^{m+n} \sin \theta A_{m+1,n}^{(0)}(\cos \theta) \]

and the coefficients \( z_{m,n} \) and \( y_{m,n} \) are

\[ z_{m,n} = j \rho \frac{a_{m,n}^{(0)} a_{m,n}^{(1)}}{a_{m,n}^{(1)} a_{m,n}^{(1)}} \]

\[ y_{m,n} = j \rho \frac{a_{m,n}^{(0)} a_{m,n}^{(0)}}{a_{m,n}^{(1)} a_{m,n}^{(1)}} \]

where \( \rho \) is the density of the spherical layer, and \( \theta_{AB} \) is the phase difference between the two spheres.

The series in equation (16) are infinite and must therefore be truncated to a finite number of terms \( n \). Hence the system of equations may be written in a matrix form as

\[ A = L + T A \]

where \( A \) and \( L \) are column matrices and \( T \) is a square matrix representing the coupling between the spheres and depends on the separation distance between the spheres.

The total scattered field in the far zone can be obtained after taking the asymptotic form of the vector spherical wave functions. Thus the total far scattered field can be written as

\[ E^{\text{sc}} = \sum_{m,n} \frac{F_{m,n}(\theta, \phi)}{\kappa^{m,n}} \]

where

\[ F_{m,n}(\theta, \phi) = F_{m,n}(\theta, \phi) + F_{m,n}(\theta, \phi) \]

and

\[ F_{m,n}(\theta, \phi) = \sum_{m,n} j^{m,n} c_{m,n} [A_{m,n}^{(0)}(\cos \theta) - B_{m,n}^{(0)}(\cos \theta)] \sin \theta \]

\[ + B_{m,n}^{(1)}(\cos \theta) \cos \theta \]

where \( c_{m,n} \) are the Neumann numbers (1 for \( m=0 \) and 2 for \( m>0 \)).

The expressions for \( F_{m,n} \) and \( F_{m+1,n} \) are obtained from those \( F_{m,n} \) and \( F_{m+1,n} \) by replacing \( A_{m,n} \), \( A_{m,n}^{(0)} \), by \( B_{m,n} \), \( B_{m,n}^{(0)} \), and multiplying each expression by the phase factor \( e^{i \theta_{AB}} \).

The normalized bistatic cross section is given by

\[ \sigma(\theta, \phi) = \frac{1}{\pi} \left[ \left| F_{m,n}(\theta, \phi) \right|^2 + \left| F_{m,n}(\theta, \phi) \right|^2 \right] \]

To obtain the bistatic cross section in the E and H planes one substitutes \( \phi=\pi/2 \) and \( \phi=\pi/2 \), respectively, in equation (25).

RESULTS

Typical numerical results are presented graphically in Fig. 2a for the normalized bistatic cross section patterns of two identical dielectric-coated spheres with \( k = 2 \), \( \omega = 1 \), \( k d = 4 \), with \( e_{i} = 5 \), as a function of the scattering angle \( \theta \) and with endfire plane wave incidence (\( \alpha = 0 \)). Fig. 2b shows the same geometry except \( k d \) is increased from 4 to 8. It can be seen that by increasing \( k d \) from 4 to 8 the magnitude of the forward scattering cross section \( (\theta = 0) \) is increased from 11.3 to 27.5, and the bistatic cross section patterns vanish at more scattering angles. Figs. 3a and 3b present the same geometry and electrical separations as in the above example except the dielectric-coatings have permittivities \( e_{i} = 5 \) and \( e_{i} = 2 \). By reducing \( e_{i} \) from 5 to 2 the ripples are substantially reduced and the magnitude of the backscattering cross section \( (\theta = 0) \) decreases from 0.2 to 0.5. This is in contrast with Fig. 3b which shows reduction in the forward scattering cross section and only a slight change in the backscattering cross section.

In the cases considered, the system of matrices is solvable only for the azimuthal mode \( m = 1 \), due to the symmetry with respect to the \( z \)-axis, with \( n = 16 \) in the case when the spheres are in contact (\( k d = 4 \)).

DISCUSSION

In this paper we have obtained an exact solution of the problem of multiple scattering by two dielectric-coated conducting spheres with arbitrary size, and angle of incidence. The boundary conditions are imposed on the outer surface of each dielectric layer by using the translation addition theorem for the spherical wave functions. The resultant system of equations is written in a
matrix form and therefore the desired field scattering coefficients are obtained by matrix inversion. Some numerical results are presented for the normalized bistatic cross section patterns for the special case of endfire incidence on two identical spheres with $ka=2$, $kb=1$ for various $kd$ and $\varepsilon_r$.

REFERENCES


Figure 1  A system of two dielectric-coated conducting spheres