ITERATIVE SOLUTION OF THE SCATTERING BY A SYSTEM OF MULTILAYERED DIELECTRIC SPHERES

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INTRODUCTION

The scattering by a single dielectric sphere with a concentric spherical shell has been analyzed in detail by a number of authors. It was firstly treated by Aden and Kerker [1] by expanding the exterior and interior fields in terms of the spherical vector wave functions. Later, Scharfman [2] used this solution to present numerical results for the special case of a small dielectric-coated conducting sphere. A solution for a conducting sphere coated with two concentric dielectric layers was given by Plonus [3]. It was found out from these studies that the presence of dielectric layers could lead to a substantial increase in the backscattering cross section for an appropriate choice of the dielectric constant and the thickness of the layers.

Up to now there is no exact analytical treatment to the problem of scattering of plane electromagnetic waves by a system of more than one dielectric sphere with concentric spherical shells. In this paper we study the effect of the dielectric layers on the multiple scattering behaviour for a system of two dielectric spheres with concentric shells. The solution to this problem is a first step towards the more general solution for arbitrary configurations of multilayered spheres which has potential applications in the simulation of complex bodies and in the electromagnetic shielding for apertureless spherical cavities.

FORMULATION OF THE PROBLEM

Figure 1 shows the system geometry. The two spheres A and B are located along the z-axis and centered at the origins O and O', respectively. The separation distance between the centers of the spheres is d. A plane electromagnetic wave with an electric field intensity of unit amplitude, is incident on the spheres at an angle α with respect to the z-axis, the plane of incidence being the x-z plane. The incident electric field is assumed to be polarized in the \( \hat{\Phi} \) direction. The incident electric and magnetic field intensities can be expressed in terms of the vector spherical wave functions of the first kind [4], i.e.,

\[
E_A^i(r, \theta, \phi) = \sum_{m=-n}^{n} \sum_{n=1}^{\infty} [P(m,n) N_{mn}^{(1)}(r, \theta, \phi) + Q(m,n) M_{mn}^{(1)}(r, \theta, \phi)]
\]  

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\[
\eta \mathbf{H}_A^s(r, \phi, \theta) = j \sum_{m=-n}^{\infty} \sum_{n=1}^{m} \left[ P(m, n) \mathbf{M}_{m,n}^{(1)}(r, \phi, \theta) + Q(m, n) \mathbf{N}_{m,n}^{(1)}(r, \phi, \theta) \right]
\]

(2)

where \( P(m, n) \) and \( Q(m, n) \) are the incident field expansion coefficients. The scattered electric and magnetic fields from sphere \( A \) \((r>a_1)\) are expanded in terms of spherical vector wave functions of the third kind in the form

\[
\mathbf{E}_A^s(r, \phi, \theta) = \sum_{m=-n}^{\infty} \sum_{n=1}^{m} \left[ A_{E}^s(m, n) \mathbf{N}_{m,n}^{(3)}(r, \phi, \theta) + A_{M}^s(m, n) \mathbf{M}_{m,n}^{(3)}(r, \phi, \theta) \right]
\]

(3)

\[
\eta \mathbf{H}_A^s(r, \phi, \theta) = j \sum_{m=-n}^{\infty} \sum_{n=1}^{m} \left[ A_{E}^s(m, n) \mathbf{M}_{m,n}^{(3)}(r, \phi, \theta) + A_{M}^s(m, n) \mathbf{N}_{m,n}^{(3)}(r, \phi, \theta) \right]
\]

(4)

where \( A_{E}^s \) and \( A_{M}^s \) are the unknown scattered field coefficients to be determined. The fields in region I \((b_1<r<a_1)\) can be written similarly in terms of the vector spherical wave function of the first and third kinds, while the fields in region II \((r<b_1)\) can be written in terms of vector spherical wave functions of the first kind only.

The scattered field from the sphere \( A \) in the presence of the sphere \( B \) is due to the incident field and to the outgoing scattered field from \( B \) \((E_0^B, H_0^B)\). In order to impose the boundary conditions at \( r=a_1 \), the outgoing scattered fields \( E_0^B \) and \( H_0^B \) are transformed into incoming fields in terms of the coordinates attached to the sphere \( A \) by applying the translation addition theorem for spherical vector wave functions [5].

**ITERATIVE BOUNDARY CONDITION SOLUTION**

The first order scattered fields result from the excitation of the spheres \( A \) and \( B \) by the incident plane wave alone. The second order field scattered by the sphere \( A \) results from the excitation of this sphere by the field scattered from the sphere \( B \) due to the initial plane wave incident field. The boundary conditions require the continuity of the total tangential electric and magnetic field intensities, i.e.,

\[
\hat{r} \times [\mathbf{E}_B^s + \mathbf{E}_A^s] = \hat{r} \times \mathbf{E}_A^s \quad \text{at} \quad r = a_1
\]

(5)

\[
\hat{r} \times [\mathbf{H}_B^s + \mathbf{H}_A^s] = \hat{r} \times \mathbf{H}_A^s \quad \text{at} \quad r = a_1
\]

(6)

\[
\hat{r} \times \mathbf{E}_A^s = \hat{r} \times \mathbf{E}_B^s \quad \text{at} \quad r = b_1
\]

(7)

\[
\hat{r} \times \mathbf{H}_A^s = \hat{r} \times \mathbf{H}_B^s \quad \text{at} \quad r = b_1
\]

(8)

The second order scattered field coefficients for the spheres \( A \) and \( B \) can be written in a matrix form as

\[
\begin{bmatrix}
\bar{A}_E^s \\
\bar{A}_M^s
\end{bmatrix}
= \begin{bmatrix}
[v_A] & 0 \\
0 & [u_A]
\end{bmatrix}
\begin{bmatrix}
[A] & [B] \\
[B] & [A]
\end{bmatrix}
\begin{bmatrix}
\bar{B}_E^s \\
\bar{B}_M^s
\end{bmatrix}
\]

(9)

\[
\begin{bmatrix}
\bar{B}_E^s \\
\bar{B}_M^s
\end{bmatrix}
= \begin{bmatrix}
[v_B] & 0 \\
0 & [u_B]
\end{bmatrix}
\begin{bmatrix}
[A'] & -(B') \\
(B') & -(A')
\end{bmatrix}
\begin{bmatrix}
\bar{A}_E^s \\
\bar{A}_M^s
\end{bmatrix}
\]

(10)

where \([v_A], [u_A], [v_B], \text{ and } [u_B]\) are diagonal matrices containing the scattering coefficients of a single sphere, while \([A], [B], [A'], \text{ and } [B']\) are square submatrices whose elements are the translation coefficients \(A_{mn}^{m' n'} \text{ and } B_{mn}^{m' n'}\) [5]. Equations (9) and
(10) can be rewritten as
\[ A_i^z = T_A \overline{B}_i \]  
\[ \overline{B}_i^z = T_B \overline{A}_i \]  
with
\[
T_A = \begin{bmatrix}
[v_A] & 0 \\
0 & [u_A]
\end{bmatrix}
\begin{bmatrix}
[A] & [B] \\
[B] & [A]
\end{bmatrix}
\]
\[
T_B = \begin{bmatrix}
[v_B] & 0 \\
0 & [u_B]
\end{bmatrix}
\begin{bmatrix}
[A'] & -[B'] \\
[B'] & -[A']
\end{bmatrix}
\]

The general expression for the ith order scattered field coefficients of spheres A and B is, therefore,
\[ \overline{A}_i^z = T_A \overline{B}_{i-1} \]  
\[ \overline{B}_i^z = T_B \overline{A}_{i-1} \]  

RESULTS AND DISCUSSIONS

Figure 2 shows the normalized backscattering cross section as a function of the separation distance, for two identical dielectric spheres with concentric spherical shells. The backscattering cross section pattern is plotted for the case of an incident plane wave at end-fire (\( \alpha = 0 \)). Results obtained with one, two and four iterations (i=1,2,4) are compared in Figs. 2 and 3 with those obtained by using the simultaneous boundary conditions solution (SBCS) [4]. Since the iterative procedure does not require matrix inversion and only a small number of iterations is needed (two to four for the range of parameters considered), the method presented in this paper is proved to be highly efficient.

REFERENCES

Fig.1. System geometry.

Fig.2. Normalized backscattering cross section versus $kd$ for two identical layered spheres with $ka=1.5$, $kb=1.0$, $\varepsilon_{r/A}=5.0$ and $\varepsilon_{r/A}=3.0$.

Fig.3. Normalized bistatic cross section versus $\theta$ for two identical layered spheres with $ka=1.5$, $kb=1.0$, $kd=4.0$, $\varepsilon_{r/A}=5.0$ and $\varepsilon_{r/A}=3.0$. 