TRANSIENT CURRENT DISTRIBUTION ON A CYLINDER ILLUMINATED BY A PLANE WAVE

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INTRODUCTION

Transient scattering of an impulsive plane electromagnetic wave by a perfectly conducting circular cylinder has been studied extensively and results are available in the literature [1,2]. Expressions have been obtained for early and late times, which are represented by separate formulas. Since the early-time solution is only valid for a very short time period immediately after the arrival of the wavefront and the late-time solution is valid after the wavefront has completely passed the cylinder, the intermediate time interval is too large for some angular region to obtain accurate results via simple interpolations of the early- and late-time results. In this paper, we derive a single formula which is mathematically valid for all the time ranges and rapidly convergent for the time range after the instant the incident wavefront has passed the cylinder axis.

ANALYTICAL EXPRESSIONS

An infinitely long, perfectly conducting circular cylinder of radius \( a \), illuminated by a normally incident plane electromagnetic wave with its electric field polarized parallel to the cylinder axis, is shown in Fig. 1. The incident electric field is given by

\[
E_i^e(r, \phi, t) = \frac{a}{c} \mathbf{E} \delta(t + \frac{r}{c} - \frac{a}{c})
\]

where \( c \) is the speed of light in free space, \( \mathbf{E} \) the characteristic impedance of free space, and \( \delta \) the Dirac delta function. The Laplace transform of the induced current density on the cylinder surface can be derived as

\[
J_z(\phi, s) = e^{-\zeta} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi \frac{1}{\zeta K_n(\zeta)}
\]

where \( \epsilon_n = 1 \) for \( n = 0 \) and \( \epsilon_n = 2 \) for \( n \neq 0 \), \( \zeta = sa/c \) with \( s \) being the Laplace transform variable and \( K_n \) is the modified Bessel function of the second kind and order \( n \). Taking the inverse Laplace transform of (2), we have

\[
J_z(\phi, t) = \frac{c}{a} \sum_{n=0}^{\infty} \epsilon_n \cos n\phi h_n(t)
\]

where \( \tau = ct/a \) and

\[
h_n(\tau) = \frac{1}{2\pi j} \int_{\zeta_0-j\infty}^{\zeta_0+j\infty} H_n(\zeta, \tau) d\zeta
\]
in which
\[ H_n(\zeta, \tau) = \frac{e^{\zeta(\tau-1)}}{\zeta K_n(\zeta)} \]  \tag{5}

In order to evaluate (4), we chose the contour as shown in Fig. 2. Hence
\[ h_n(\tau) = -\frac{1}{2\pi i} \left[ \oint_{C_\kappa} + \int_{L_1} + \int_{L_2} \right] H_n(\zeta, \tau) d\zeta + \sum_l R_{nl} \]  \tag{6}

where \( R_{nl} \) is the residue of \( H_n(\zeta, \tau) \) at the pole \( \zeta_{nl} \) at which \( K_n(\zeta) = 0 \). It can be shown that
\[ \lim_{\xi \to 0} \int_{C_\kappa} H_n(\zeta, \tau) d\zeta = 0 \]  \tag{7}

and when \( \tau > 0 \)
\[ \lim_{R_\to \infty} \int_{C_\kappa} H_n(\zeta, \tau) d\zeta = 0 \]  \tag{8}

Using appropriate analytic continuation of the modified Bessel functions, the contributions from the line integrals along the branch cut can be evaluated as
\[ \int_{L_1} H_n(\zeta, \tau) d\zeta + \int_{L_2} H_n(\zeta, \tau) d\zeta = -2\pi j B_n(\tau) \]  \tag{9}

where
\[ B_n(\tau) = \int_0^\infty \frac{I_n(y) e^{-\gamma(\tau-1)}}{y[K_n^2(y) + \pi^2 I_n^2(y)]} dy \]  \tag{10}

For the residue terms, since the zeros of \( K_n(\zeta) \) appear in conjugate pairs, \( \sum_l R_{nl} \) can be obtained as
\[ \sum_l R_{nl} = -2^{[n/2]} \sum_{l=1}^{[n/2]} \text{Re} \left[ \frac{e^{\zeta_{nl}(\tau-1)}}{\zeta_{nl} K_{n+1}(\zeta_{nl})} \right] \]  \tag{11}

where \([n/2]\) represents the integer part of \(n/2\). From (3) - (11), we finally obtain
\[ J_z(\phi, \tau) = \frac{e}{\alpha} \sum_{n=0}^\infty e_n \cos n\phi \left[ B_n(\tau) + \sum_l R_{nl} \right] \quad \text{for} \ \tau > 0 \]  \tag{12}

with \( B_n(\tau) \) and \( \sum_l R_{nl} \) given by (10) and (11), respectively.

**NUMERICAL RESULTS AND DISCUSSION**

Numerical results for the induced current density are presented in Fig. 3 where \( c_t/d = (\tau - 1)/2 \) with \( d = 2a \). The results calculated from (12) are normalized to \( 20\pi a/c \) in order to compare with those in [1]. It should be noted that the results in [1] are calculated with three different formulas corresponding to the early-time approximation, the late-time approximation and an approximation for intermediate times based on using an interpolation between the early- and the
late-time solutions. It can be seen from Fig. 3 that the results from the single expression (12) cover all the time ranges in the shadow region. They also cover the late time range in the illuminated region and part of the early and intermediate time ranges for large angles in this region. This shows that expression (12) has a significantly larger range of validity than the late-time approximation given in [1,2].

REFERENCES


Fig. 1. Cross section of the cylinder illuminated by an impulsive plane wave.

Fig. 2. Integration contours in the complex $\zeta$-plane.
Fig. 3. Surface current density at $\phi = (a) 90^0$, (b) $120^0$, (c) $150^0$, (d) $180^0$.  